THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. DEGREE PROGRAMME 2006/2007

FINAL EXAMINATION 2007

PHU 3148 / PHE 5148 MATHEMATICAL PHYSICS

DURATION : TWO AND HALF HOURS (2 1/2 HRS)

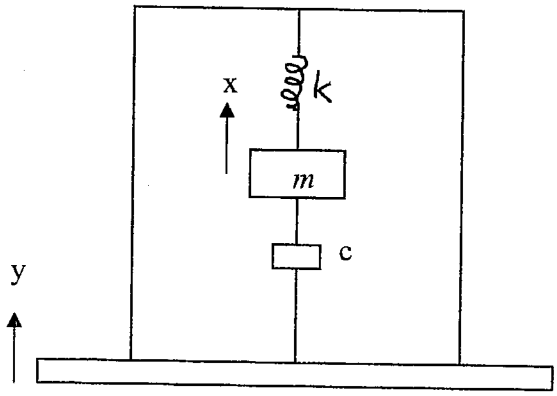
Date : 21 - 04 - 2007 **Time : 1.30 pm – 4.00 pm**

Answer **FOUR** Questions

1. (a) A particle of mass m moves on a rotating disc towards the edge. Its position vector is given by $\vec{\mathbf{r}} = bt(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$, where ω is the angular speed of the disc rotating anticlockwise. Find the acceleration $\frac{d^2\vec{\mathbf{r}}}{dt^2}$ of the particle. If $\vec{\mathbf{F}} = m\frac{d^2\vec{\mathbf{r}}}{dt^2}$ calculate the instantaneous power $p = \frac{d}{dt}(\vec{\mathbf{F}}.\vec{\mathbf{r}})$.

   (b) Prove the product rules
   $\nabla \times (f\vec{\mathbf{A}}) = f(\nabla \times \vec{\mathbf{A}}) - \vec{\mathbf{A}} \times (\nabla f)$ and $\nabla.(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) = \vec{\mathbf{B}}.\ (\nabla \times \vec{\mathbf{A}}) - \vec{\mathbf{A}}.\ (\nabla \times \vec{\mathbf{B}})$
   where, f is a scalar and, **A** and **B** are vectors.
   The magnetic field at a point P = (x,y,z), produced by a current carrying rod of uniform current density **J** is given by $\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{\vec{\mathbf{J}} \times \vec{\mathbf{r}}}{r^3}$.
   Find **r** and r^3.
   Using the above product rules show that $\nabla.\vec{\mathbf{B}} = 0$

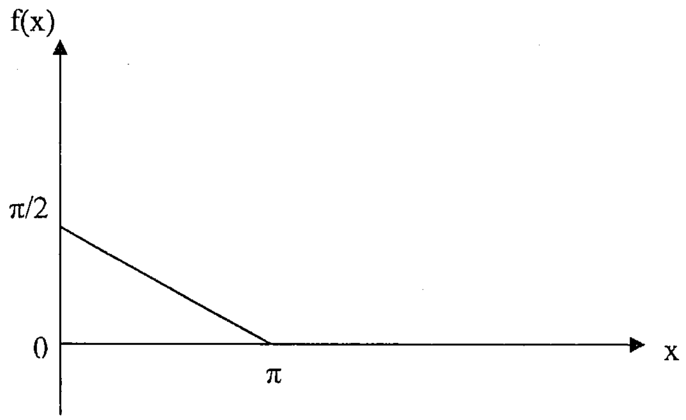
2.



The above figure shows an arrangement of a vibrating measuring instrument. The equation of motion of the mass m is given by $m\ddot{x} = -c(\dot{x} - \dot{y}) - k(x - y)$ where x and y are the displacement of the mass and the vibrating body, respectively. Assume, sinusoidal motion $y = A\ \sin\omega t$ of the vibrating body. Set $z = x - y$ and show that the above differential equation reduces to $m\ddot{z} + c\dot{z} + kz = m\omega^2 A \sin\omega t$
Find the steady state solution of z using the method of undetermined multipliers.

3. Consider the Legendre's equation $(1 - x^2)\dfrac{d^2\Theta}{dx^2} - 2x\dfrac{d\Theta}{dx} + n(n+1)\Theta = 0$, where n is a positive integer. Show that the above differential equation posses the general solution $\Theta(x) = a_0 \sum_{k=0,2,4..}^{\infty} c_k x^k + a_1 \sum_{k=1,3,5..}^{\infty} c_k x^k$.
Find out the first five values of c_k (ie.,for $k = 0,1,2,3,4,5$)
Set $a_0 = (-1)^{n/2} \dfrac{n!}{2^n \left[(n/2)!\right]^2}$ and $a_1 = (-1)^{(n-1)/2} \dfrac{(n+1)!}{2^n[(n-1)/2]![(n+1)/2]!}$
Find the series for $n = 0,1,2,3,4,5$ (These series solutions for different values of n are called Legendre polynomials $P_n(x)$) Show that $P_n(1) = 1$.

4. Explain the idea of half range expansion of a given function using Fourier sine or cosine series.

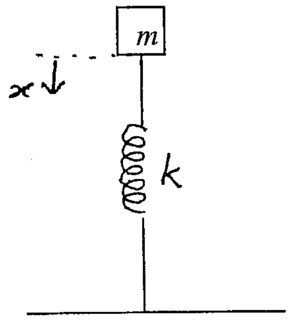


Consider the function f(x) given in the above figure. Using a suitable half range expansion of the function show that $f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \sin nx$

Establish the result: $\sum_{n=1}^{\infty} (1/n^2) = \pi^2/6$

5. The wave equation for longitudinal waves traveling in a rod with both ends free is given by $\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}$ where, E is the Young's modulus and ρ is the density of the rod. Here the speed of propagation of longitudinal waves in the rod is $c = \sqrt{\frac{E}{\rho}}$. Assume that the boundary conditions for the problem are $\frac{\partial u}{\partial x} = 0$ at $x = 0$ and $x = l$. Show that with zero initial displacement of the rod the amplitude of the longitudinal vibrations along the rod is $u = u_0 \cos\frac{n\pi x}{l} \sin(\frac{n\pi}{l}\sqrt{\frac{E}{\rho}}t)$ and the frequency of vibration is $f = \frac{n}{2l}\sqrt{\frac{E}{\rho}}$ where $n = 1,2,3\ldots$.

6.



A mass m is dropped from rest through a height h on to a vertical spring as shown in the figure. The equation of motion of the spring mass system is $m\ddot{x} + kx = mg$.

Assume the initial conditions $x(0) = 0$ and $\dot{x}(0) = \sqrt{2gh}$. Solve the above differential equation using Laplace Transform method and show that

$$x(t) = \sqrt{\frac{2gh}{\omega^2} + \left(\frac{g}{\omega^2}\right)^2}\,\sin(\omega t - \phi) + \frac{g}{\omega^2} \text{ where } \tan\phi = \frac{1}{\omega}\sqrt{\frac{g}{2h}}.$$

Prove any formula you may use.