The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2017/2018
Applied Mathematics – Level 05

ADU5303/APU 3145– Newtonian Mechanics II

Duration :- Two Hours

Date :- 06.04.2019 **Time:-01.30 p.m. to 03.30 p.m.**

Answer Four Questions Only.

1. (a) In the usual notation, show that in cylindrical polar coordinates, the velocity and acceleration of a particle are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$, respectively.

   (b) A particle of mass m moves on the smooth inside surface of a parabola of revolution $\rho^2 = 4az$, whose axis is vertical with vertex downwards. The path of the particle lies exactly between $z = \alpha$ and $z = \beta$. Show that the angular momentum of the particle about the z-axis is $m(8az\alpha\beta)^{\frac{1}{2}}$ and that its speed is $(2g(\alpha + \beta - z))^{\frac{1}{2}}$. Also, show that the reaction between the particle and the surface at $z = \alpha$ is $\dfrac{mg(a+\beta)}{(a(\alpha+\beta))^{\frac{1}{2}}}$.

2. (a) In the usual notation, show that in spherical polar coordinates, the velocity and acceleration of a particle are given by $\underline{\dot{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\underline{k}$ and

$$\underline{\ddot{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\,\dot{\phi}\right)\hat{\phi}$$

   respectively.

   (b) A particle P moves on the outside surface of a smooth circular cone of half angle α, fixed with its axis vertical and vertex O, uppermost. Initially P is projected horizontally along the cone with speed u from the point A, where $OA = a$. Find the reaction on the particle, when $OP = r$. Find a condition on u, α and a such that the particle remains on the cone throughout.

3. (a) Obtain, in the usual notation, the equation $\dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \dfrac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) An object is projected vertically downward with speed v_0. Prove that after time t, the object is deflected east of the vertical by the amount $\omega v_0 \cos\lambda t^2 + \frac{1}{3}\omega g t^3 \cos\lambda$ where λ is the latitude of the point of projection and ω is the angular speed of the earth about its polar axis.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2\ldots, n.$

(b) A homogeneous rod OA of mass m_1 and length $2a$ is freely hinged at O to a fixed point. Another homogeneous rod AB of mass m_2 and length $2b$ is freely hinged at A to the free end of the rod OA. If the system moves in a fixed vertical plane, show that the Lagrangian is given by

$$L = 2a^2\dot{\theta}^2\left(\frac{m_1}{3} + m_2\right) + \frac{2}{3}b_1^2 m_2\dot{\phi}^2 + 2m_2 ab\dot{\theta}\dot{\phi}\cos(\phi - \theta) + m_1 ga\cos\theta + m_2 g(2a\cos\theta + b\cos\phi).$$

Hence obtain the equations of motion of the two rods, using Lagrange's equations of motion.

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) If a rectangular parallelepiped with its edges $2a, 2a, 2b$ rotates about its center of gravity under no forces. Prove that, its angular velocity about one principal axis is a constant n and is periodic about other axis, the time period being $2\pi(a^2 + b^2)/[(b^2 - a^2)n]$.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2,$$

where a, b, k_1 and k_2 are constants. Find the Hamiltonian for the system and hence write down equations of motion in Hamiltonian formulation.