The Open University of Sri Lanka
B.Sc Degree Programme
Level 05 - Final Examination 2007/2008
Pure Mathematics
PMU 3292/PME 5292 – Group Theory and Transformation - Paper I

065

Duration :- 2 ½ Hours.

Date :- 29–12–2007. **Time:- 9.30 am. – 12.00 noon.**

Answer FOUR questions only.

1.(a) Prove that the set of the six special bilinear mappings $f_1, f_2, f_3, f_4, f_5, f_6$ defined by $f_1(z) = z$, $f_2(z) = \frac{1}{z}$, $f_3(z) = 1 - z$, $f_4(z) = \frac{z}{z-1}$, $f_5(z) = \frac{1}{1-z}$, $f_6(z) = \frac{z-1}{z}$ of the infinite complex plane into itself is a finite non–abelian group of order 6.

(b) If (G, o) is group, then prove the followings

(i) The identity element of G is unique.

(ii) Every $a \in G$ has a unique inverse, a^{-1} in G.

02. Show that the matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}$$

where $\omega^3 = 1$, $\omega \neq 1$ form a group G under the matrix multiplication. Also, obtain all abelian subgroups of this group G.

Are there any non–abelian subgroups of G? Justify your answer.

03. (a) A group G has the property that for each element a of G, $a^2 = e$, where e is the identity of G. Prove that G is abelian.

(b) A group G has independent generators a, b

such that $a^2 = b^2 = (ab)^2 = I$.

Show that G is of order 4 and construct its multiplication table.

Also, show that $ab = ba$.

04. (a) Prove that the set $\{I, (1, 2), (3, 4), (1\ \ 2)(3\ \ 4)\}$ is a subgroup of S_4, where S_4 is the symmetric group on the set $\{1, 2, 3, 4\}$.

(b) Let G be an abelian group. Let H be the subset of G consisting of those elements $x \in G$ such that $x = x^{-1}$ that is

$$H = \{x \in G : x = x^{-1}\}.$$

Prove that H is a subgroup of G.

05. (a) If G is the group of permutations on the set $\{1, 2, 3\}$ show that

$$s = (1\ \ 2\ \ 3),\ t = (1\ \ 2) \text{ satisfy the equations } s^3 = t^2 = (st)^2 = e,$$

where e is the identity of G.

Show also that $st = ts^2$.

(b) Express the following permutations as a product of disjoint cycles.

(i) $(4\ \ 2\ \ 1\ \ 5)(3\ \ 4\ \ 2\ \ 6)(5\ \ 6\ \ 7\ \ 1)$

(ii) $(1\ \ 3\ \ 2\ \ 6)(1\ \ 2\ \ 4)(3\ \ 5)$.

06. (a) Find all the elements of $(Z_{12}, +)$ of order 2, 3, 4, 6 and 12.

(b) Prove that, the group $(Z_{11}^{*}, \times)$ is cyclic.

Also, find all the generators of the group $(Z_{11}^{*}, \times)$.