00100

THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering / Bachelor of Software Engineering Honors

Final Examination (2016/2017)
MPZ4140 /MPZ4160: Discrete Mathematics I

Date: 27th November 2017 (Monday) **Time: 9:30 am – 12:30 pm**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Decide which of the following are propositions: **[20%]**

a) "$x + 5 > x + 3$ ";
b) "the cat has no five legs" ;
c) "if $9 - 15 \neq 7$ then, $12 + 3 = 17$ and $6 - 3 = 3$";
d) "$y \leq 3$".

II. State the "convers", "inverse", and "contrapositive" of each of the following statement: **[30%]**

a) If x is an even integer, then x^2 is even;
b) If the product of two integers x and y is odd, then both x and y are odd.

III. Let p, q, and r be three statements.
Verify that $[\sim(p \wedge q) \vee r] \to [p \to \sim(q \wedge \sim r)]$ is a tautology or not. **[20%]**

IV. Let m be the proposition "Chandana passes calculus"; let n be the proposition "Chandana is happy"; and let l be the proposition "Chandana has the job." Write out the following propositions in words: **[20%]**

a) $m \to n$;
b) $n \to (m \wedge l)$;
c) $\sim(m \leftrightarrow n)$;
d) $l \vee (m \to n)$.

V. Show that $p \vee (p \wedge q) \leftrightarrow p$ using laws of the algebra of propositions. **[10%]**

Q2.

I. Give the negation of the following statements: **[20%]**

a) $\forall x[x^2 > 0]$;
b) $\exists x\,[2x = 1]$;
c) $\forall x\,\exists y\,[x + y = 1]$;
d) $\forall x\,\forall y\,[x > y \Rightarrow x^2 > y^2]$.

II. Test the validity of the following arguments:

a) I study hard if and only if I get rich
I rich.

Therefore I study hard **[15%]**

b) Pasindu bought a personal computer or a video cassette recorder (VCR).
If he bought a VCR, then he likes to watch movies at home.
He does not like to watch movies at home.

Therefore Pasindu bought a personal computer. **[25%]**

III. Prove De Morgan's laws for propositions by using truth tables. **[10%]**

IV. Prove each using the method of contradiction, **[30%]**

a) If the square of an integer is even, then the integer is even.
b) $\sqrt{2}$ is an irrational number.

Q3.

I. Prove that for all integer n, n is odd if and only if $n - 1$ is even. **[20%]**

II. Using Mathematical induction, for a positive integer n, prove that each of the following: **[45%]**

a) $7^n - 1$ is divisible by 6 for all $n \geq 1$;
b) $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

III. Prove directly that the product of any two odd integers is an odd integer. **[15%]**

IV. By giving a counter example, disprove each of the following statements:

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ x - y^2 = 19$. **[10%]**
b) $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \sqrt{xy} \leq \frac{(x+y)}{2}$. **[10%]**

SECTION – B

Q4.

I. Write down the elements in each of the following sets: [20%]

a) $P = \{x : |x - 5| \leq 6, and\ x \in \mathbb{Z}^{+}\}$;

b) $Q = \{x: x^2 + 9 = 0, x \in \mathbb{R}\}$;

c) $R = \{x: x = 1 + (-1)^n, n \in \mathbb{Z}\}$;

d) $S = \{x: x^2 + x - 6 = 0, x \in \mathbb{N}\}$.

II. Let $A = \{a, b, c, d, e, f\}, B = \{c, d, e, f, g, h\}, C = \{f, g, h, i, j, k\}$. Find [15%]

a) $A \backslash B$;

b) $A \oplus B$;

c) $A \cup (B \oplus C)$.

III. Define the Cartesian product of set A and B. [05%]

a) $M = \{a, ab, b\}$ and $N = \{1, 12, 2\}$. Find $M \times N$ and N^2. [20%]

b) Let A, B and C be sets. Show that
$A \times (B \cup C) = (A \times B) \cup (A \times C)$. [20%]

IV. Let A, B, C are non-empty sets. Assuming that $|A \cup B| = |A| + |B| - |A \cap B|$, show that
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
[20%]

Q5.

I. Consider the following relations defined on the set of natural numbers $\mathbb{N}$. [30%]

R_1: "x is a multiple of y";
R_2 : " $a + 3b = 12$";

State whether or not each of the relations

a) reflexive

b) symmetric

c) transitive

II. Let A be a set of nonzero integers and let "$\sim$" be the relation on $A \times A$ define by

$$(a, b) \sim (c, d) \text{ whenever } ad = bc.$$

Prove that "$\sim$" is an equivalence relation on A. [35%]

III. Prove that following relation is an equivalence relation and describe the equivalence classes.

The relation $\boldsymbol{mR_3n} \Leftrightarrow \boldsymbol{m^2 - n^2}$ is divisible by 3 on the set $\mathbb{Z}$. [35%]

Q6.

I. Consider the function $f(x) = x^2 - 3x + 2$. Find

a) $f(x+h)$, **[05%]**

b) $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$. **[10%]**

II. Define a one to one and onto function. **[10%]**
Check whether the following functions are one to one and onto.

a) $f: \mathbb{R} \to \mathbb{R}$ and $f(x) = |x|$. **[15%]**

b) $g: \left[0, \frac{\pi}{2}\right] \to [0,1]$ and $g(x) = \sin x$. **[15%]**

III. Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{2/3\}$. Define $f: A \to B$ by $f(x) = \frac{2x+6}{3x-15}$.

Prove that $f(x)$ is invertible and find the formula of $f^{-1}(x)$. **[35%]**

IV. Let $f(x) = ax + b$ and $g(x) = \frac{x-b}{a}$ on $\mathbb{R}$, where $a \neq 0$. Find $g \circ f$ and $f \circ g$. **[10%]**

SECTION – C

Q7.

I. Given integers $a, b, c,$ and d, prove that,

a) if $a|b$, $a|c$ and $a|d$, then $a|(2b + c - 3d)$, **[15%]**

b) if $a|b$ and $b|a$, then $a = \pm b$, **[15%]**

c) if $a|b$, then $a|\ |b|$, **[15%]**

d) if $a|b$ and $b|c$, then $a|dc$. **[15%]**

II. Prove that if $x \in \mathbb{Z}^+$ and $(x-1)|\ (x^2 + 3x - 4)$,
then $(x^2 - 1)|\ (3x^3 + 12x^2 - 3x - 12)$. **[20%]**

III. If b $(b \neq 2)$ is a prime number, show that
$b^2 + (b+2)^2 + (b+4)^2 + 1$ is divisible by 12. **[20%]**

Q8.

I. Let $a, b, c, d \in \mathbb{Z}$. Show that

a) if $\gcd(a, c) = \gcd(b, c) = 1$, then $\gcd(ab, c) = 1$, [20%]

b) if $\gcd(a, b) = d$, then $\gcd\left(a/_d, b/_d\right) = 1$, [10%]

c) if $a|c$ and $b|c$, with $\gcd(a, b) = 1$, then $ab|c$, [20%]

II. Find the $\gcd(1769, 2376)$, and express it as $1769\,x + 2376\,y = \gcd(1769, 2376)$ by using the Euclidean Algorithm, and determine integers m and n of the following equation:
$1769\,m + 2376\,n = 65.$ [50%]

Q9.

I. Let $a \equiv b(mod\ n)$ and $b \equiv c(mod\ n)$. Show that $a \equiv c(mod\ n)$. [15%]

II. If $a \equiv b(mod\ m)$ and $c \equiv d(mod\ m)$, then show that $ac \equiv bd(mod\ m)$. [15%]

III. Let $a \equiv b(mod\ m)$ and $n \in \mathbb{Z}^+$. The show that $a^n \equiv b^n(mod\ m)$. [20%]

IV. Solve the following system of congruence:

$$2x \equiv 3(mod\ 5)$$
$$3x \equiv 4(mod\ 7)$$
$$5x \equiv 7(mod\ 11).$$

[50%]