

THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. DEGREE PROGRAMME 2007/2008

FINAL EXAMINATION 2007

PHU 3142 / PHE 4142 PHYSICAL BASIS OF QUANTUM THEORY, THEORY  
OF RELATIVITY AND SOLID STATE PHYSICS

DURATION : TWO & HALF HOURS (2 1/2 HR)

Date : 18 - 12 - 2007

Time : 1.30 - 4.00 pm

( $m_e = 9.11 \times 10^{-31}$  kg,  $e = -1.6 \times 10^{-19}$  C,  $h = 6.63 \times 10^{-34}$  Js)

Answer FOUR Questions

1. State de Broglie hypothesis.  
A photon beam of wavelength  $\lambda$  is scattered by a free electron of mass  $m_e$ . If the beam is scattered at angle  $\phi$  show that its wavelength is increased by  $\frac{h}{m_e c}(1 - \cos \phi)$ . Show that the maximum kinetic energy of the recoiling electron is  $\frac{hc}{\lambda(1 + \frac{mc\lambda}{2h})}$ . What is the maximum kinetic energy of electrons knocked out of a thin copper foil by Compton scattering of an incident beam of 17.5 keV x-rays?
- 2.(a) Consider a free particle wave packet defined below:  

$$\psi(x,0) = A \sin(3\pi x / L) \quad |x| \leq L$$

$$\psi(x,0) = 0 \quad |x| > L$$
 Calculate A and  $\langle x \rangle$   
 Calculate the mean square deviation in the particle's position,  $\langle (x - \langle x \rangle)^2 \rangle$
- (b) An electron is trapped in a two dimensional infinite square potential well with widths  $L_x = L_y = L$ . Show that the energy of the electron is given by  $E_{n,l} = \frac{h^2}{8m} (\frac{n^2}{L^2} + \frac{l^2}{L^2})$  where  $n$  and  $l$  are quantum numbers.  
 Find the energies of the lowest five energy levels for the electron and construct an energy level diagram.
3. Using Lorentz transformation equations show that the simultaneity is not possible between two reference frames.  
 A spaceship is passing by a planet - moon system in the universe. It detects a high energy microwave burst at a location in the moon and 1.1s later an explosion at the planet. The distance between the burst and the explosion is  $4 \times 10^8$  m as

measured from the spaceship's reference frame. The speed of the spaceship relative to the planet and its moon is  $0.98c$ .

- (a) What are the distance and time interval between the burst and the explosion as measured in the planet-moon inertial frame? Comment on your results.
- (b) Show that the burst and the explosion are two unrelated events as discovered by two inertial frames. (Hint: If they are related information must travel from the location of one event to that of the other.)

4. Show that the kinetic energy  $K$  of a relativistic particle of mass  $m$  is

$$K = mc^2(\gamma - 1) \text{ where } \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

If the momentum of the particle is  $p$  show that  $(pc)^2 = K^2 + 2Kmc^2$ .

A proton in the cosmic rays coming to Earth has kinetic energy of  $3 \times 10^{20}$  eV.

- (a) What is the speed of the proton relative to the Earth-based detector? Assume that  $mc^2$  for the proton is 938 MeV.
  - (b) Suppose that the proton travels along the diameter ( $9.8 \times 10^4$  light years) of the Milky Way galaxy. How long does the proton take to travel that diameter as measured from the common reference frame of Earth and the galaxy?
  - (c) How long does the trip takes as measured in the reference frame of the proton?
- 5.(a) Consider mono atomic structures of sc, bcc and fcc lattice types with a cube edge  $a$ . Calculate the nearest neighbour distance and packing fraction for each type. Determine the radius of the largest atom that will fit into the place at the body centre of the bcc structure.

- (b) Show that in a cubic crystal of lattice constant  $a$  the spacing between consecutive parallel planes of Miller indices  $(hkl)$  is given by  $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

Consider a simple cubic crystal structure with cube edge  $3.5 \text{ \AA}$  and suppose it is used to scatter  $3.1 \text{ \AA}$

6. According to the Fermi-Dirac statistics what is the probability  $P(E)$  that an energy level will be occupied by an electron at the temperature  $T$ ?

Show that  $P(E_F + \Delta E) + P(E_F - \Delta E) = 1$  where  $E_F$  is the Fermi energy.

- (a) Show that for free electrons the density of states per unit volume

$$\text{is } D(E) = \frac{m^{3/2} \sqrt{2E}}{\pi^2 \hbar^3}$$

- (b) For a certain metal the conduction band is partially filled and free electron like. Show that the free electron concentration of the metal is

$$n = \frac{2\sqrt{2}(m^*)^{3/2}}{3\pi^2 \hbar^3} (E_F - E_c)^{3/2} \text{ where, } E_c \text{ is the minimum energy in the conduction}$$

band and  $m^*$  is the effective mass of the free electrons.

Al has a fcc structure with cube edge  $4.05 \text{ \AA}$ . Each atom of Al contributes 3 free electrons to the conduction band. Assuming, Fermi level is 12 eV above the bottom of the conduction band calculate the effective mass for electrons in Al.

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