The Open University of Sri Lanka
B.Sc Degree Programme
Level 05 - Final Examination 2008/2009
Pure Mathematics
PMU 3292/PME 5292 – Group Theory and Transformation - Paper I

Duration :- 2 ½ Hours.

Date :- 24–12–2008. **Time:- 9.30 a.m. – 12.00 noon.**

Answer FOUR questions only.

01. (a) Let $G=\{x+\sqrt{3}y \mid x,y\in Q\}$. Prove that $(G,+)$ is an abelian group.

(b) Let $C(n)=\{x\in C \mid x^n=1\}$, n being a positive integer and let '$\cdot$' be the multiplication of complex numbers. Show that $(C(n),\cdot)$ is a group.

(c) If a group G has an element x such that $ax=x$ for all $a\in G$, then show that $G=\{e\}$.

02. (a) Show that set $S=\{I, (1\ \ 2)(3\ \ 4), (1\ \ 3)(2\ \ 4), (1\ \ 4)(2\ \ 3)\}$ form a group under the permutation multiplication.

(b) Prove that the set of nonzero matrices $\begin{pmatrix} x & y \\ -y & x \end{pmatrix}$ where $x, y\in\mathbb{R}$ form a group under the matrix multiplication.

03. (a) Which of the following subsets of S_4 are subgroups? Justify your answer.

(i) $\{I, (1\ 2\ 3), (1\ 3\ 2)\}$

(ii) $\{I, (1\ 2\ 3), (1\ 3\ 2), (1\ 3\ 4), (1\ 4\ 3)\}$

(iii) $(I, (1\ 2\ 3\ 4), (1\ 3)(2\ 4), (1\ 4\ 3\ 2)\}$.

(b) Prove that a non empty subset H of a group G is a subgroup of G if for all $x, y\in H$, the element $xy^{-1}\in H$.

04. (a) Determine the elements of the cyclic group generated by the matrix $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$.

(b) Let a, b be elements of a group G. Assume that a has order 5 and that $a^3b=ba^3$. Prove that $ab=ba$.

(c) What is the subgroup of $(\mathbb{Z}_{18},\oplus)$ generated by $\bar{3}$?

05. (a) Express each of the following permutations as products of disjoint cycles:

(i) (1 2 3) (4 5) (1 3 4 5)

(ii) (1 2) (5 4) (3 2) (1 7) (2 8)

(iii) (4 5) (1 2 3) (3 2 1) (5 4) (2 6) (1 4).

(b) Find the order of the following elements of the given group:

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 2 & 5 & 4 \end{pmatrix} \in S_5$

(ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 1 & 6 \end{pmatrix} \in S_6$.

06. Let $a \in G$. Define $N(a) = \{x \in G \mid ax = xa\}$. Prove the following.

(a) $N(a)$ is a subgroup of G.

(b) For any two elements $x, y \in G$, $x^{-1}ax = y^{-1}ay \Leftrightarrow N(a)x = N(a)y$.