The Open University of Sri Lanka
B.Sc. /B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination - 2017/2018
Level 05 - Applied Mathematics
ADU5305/ ADE5305/APU3147/APE5147– Statistical Inference

Duration: - Two Hours.

DATE: 22-04-2019 **Time: 9.30 a.m. to 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.
Answer only four questions.

1.

A company that produces a certain electrical product claims that the life time X (in years) has the density function

$$f(x,\beta) = \beta exp(-\beta x) \quad ; \beta > 0, \; x > 0$$

and the moment generating function of X is given by

$$M_X(t) = \frac{\beta}{\beta - t}; \; t < \beta$$

Let $X_1, X_2, \ldots X_n$ denote lifetimes of n randomly chosen products from the above population.

(i) Show that the expected life time and variance of a randomly selected electrical product are β^{-1} and β^{-2} respectively.

(ii) Derive the maximum likelihood estimator for expected life time of a randomly selected electrical product. Is the estimator derived by you an unbiased estimator for expected life time of a randomly selected electrical product? Justify your answer.

(iii) Derive the maximum likelihood estimator for variance of a randomly selected electrical product.

(iv) A sample drawn from the above distribution is given below. Estimate the expected life time and variance of a randomly selected electrical product using part (ii) and part (iii).

2.29	8.65	6.74	8.19	3.77	6.27	4.29	15.72	10.78	6.50	0.48	0.87
1.90	2.04	11.97	5.89	30.70	0.03	13.44	21.84				

(v) Find the sample size necessary to estimate the expected life time of a randomly selected electrical product within error bound on six months with 95% confidence.

2.

(a) An investigation was conducted on the dust content in the flue gases of two types of solid – fuel boilers type A and type B. Thirteen boilers of type A and 9 boilers of type B were used under identical fueling and extraction conditions and over a similar period. The quantities in grams of dust deposited were recorded. Sample means of dust contents of type A boilers and type B boilers are 63.83 grams and 52.89 grams respectively. From the past experiences it is reasonable to assume that these samples come from normal populations and with equal standard deviation of 9.00 grams. Test for an equality of population means. Use 0.05% level of significance. Clearly state the findings.

(b) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a distribution with density given by $f(x;\theta)$. Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3,$ are functions of $X_1, X_2, X_3, \ldots\ldots\ldots X_n$. Suppose for large samples $\hat{\theta}_1, \hat{\theta}_2$ are accurate and precise estimators for parameter θ. $\hat{\theta}_3$ is an unbiased and the likely hood estimator for parameter θ. For small samples $MSE(\ \hat{\theta}_1\ ) < \ MSE(\ \hat{\theta}_2\ )$. State whether the following statements are true or false. Justify your answer.

(i) Always $E(\frac{\widehat{\theta_1}+\widehat{\theta_2}}{2}) = \theta$

(ii) For large samples $Var(\hat{\theta}_2) = Var(\hat{\theta}_1)$

(iii) Let $L(\theta)$be the likelihood function of θ. $L(\hat{\theta}_1) < \ L(\hat{\theta}_3)$.

(iv) $\hat{\theta}_3$ is not an accurate estimator for θ

(v) For large samples $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ are accurate and precise estimators for θ.

3.

According to past experience a company that produces a certain electrical bulb claims that the life time $\boldsymbol{X}$ (in years) has a normal distribution with unknown mean and variance. A sample drawn from the above distribution is given below.

8.16	11.50	7.28	10.07	12.90	11.01	11.11	8.27	11.65	9.68	11.46	6.26	10.39
8.05	10.48	14.62	8.96	10.72	10.37	11.71						

(i) Find 95% confidence interval for mean lifetime of a randomly selected bulb. Interpret your results.

(ii) Find 90% confidence interval for variance lifetime of a randomly selected bulb. Interpret your results.

(iii) Using a suitable statistical test comment on the claim that "mean lifetime of a randomly selected electrical bulb exceeds 10.5 years".

4.

The production manager is interested in the proportion θ of nonconforming automotive crankshaft bearings produced in a production process of automotive crankshaft bearings. Suppose total production on a day is 10000. Random sample of 80 automotive crankshaft bearings (drawn with replacement) were tested on this particular day by the production manager. Suppose that 8 of the bearings had surface finish that is rougher than the specifications will allow.

(i) Construct a 95% confidence interval for θ

(ii) Construct 95% confidence interval for the total number of nonconforming automotive crankshaft bearings in the production on that day. Hence comment on the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day is 110"

(iii) Using a suitable statistical test comment on the claim that "proportion θ of nonconforming automotive crankshaft bearings produced on that particular day is greater than 0.11"

(iv) Using Part (iii) test the validity of the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day exceeds 110"

5.

(a) Briefly explain the following terms.

(iii) Point estimation

(v) Interval Estimation

(b) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{1+\theta} \quad ; \ 0 \le x \le 1+\theta ; \quad \theta > 0$$

(i) Prove that he mean of the above distribution is $\frac{1+\theta}{2}$.

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you, an unbiased estimator for θ? Prove your answer.

(iii) Derive the maximum likelihood estimators for θ and for mean of the above distribution.

(iv) A random sample drawn from the above distribution is given in the following table.

0.49	1.38	1.95	1.28	1.80
0.21	0.05	0.48	0.49	1.40
0.59	0.23	0.26	1.65	1.43
1.56	0.16	0.82	0.37	0.80

[i] Estimate θ using moment estimator derived in part(ii).

[ii] Estimate θ and mean of the above distribution using maximum likelihood estimators derived in part(iii)

6.

(a) Briefly explain the following terms.

(i) Significance level of a statistical test

(ii) Power of a statistical test

(b) Assignment marks and final examination marks of a subject for randomly selected 15 students are given below.

Student Name	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Assignment mark	60	47	60	56	47	27	45	61	68	62	35	53	57	25	31
Final Mark	67	54	53	49	47	35	30	77	57	54	42	60	63	42	28

Using suitable statistical test, test the validity of the claim that "Expected assignment mark is greater than the expected final examination mark for a randomly selected student". Use 5% level of significance.