The Open University of Sri Lanka
B.Sc. Degree Programme/Continuing Education Programme
Final Examination 2007/2008
AMU 2185/AME 4185-Numerical Methods I
Level 04-Applied Mathematics

130

Duration: 2 ½ hours

Date: 25/06/2008. **Time: 10.00 a.m.-12.30 p.m.**

Answer four questions only

(1) (*a*) Briefly explain

(i) Percentage error

(ii) Absolute error

(iii) Relative error.

(*b*) If, $u = 6v^7 - 3v$, find the percentage error in u at $v = 1$ if the error in v is 0.05.

(*c*) Expand $\sin x$ as a power series in x up to the term in x^3 and use the approximation to calculate $\sin(0.1)$.

Hence find the limiting absolute error.

(2) (*a*) Let $f \in c[a,b]$ and suppose $f(a)f(b) < 0$. Show that the Method of Bisection generates a sequence $\{x^{(n)}\}$ approximating the solution, x^*, with the property

$$\left|x^* - x^{(n)}\right| \le \frac{1}{2^n}(b-a)\ ,\ n \ge 1 \text{ where } n \text{ is the number of iterations.}$$

(b) Estimate the number of iterations that will be required to find the solution of $\tan x + \tanh x = 0$, using the interval (2.3, 2.4) correct to 2 decimal places by the Method of Bisection.

(c) Find the real root correct to 2 decimal places of the equation $\tan x + \tanh x = 0$ in the interval (2.3, 2.4) by using the Method of Bisection.

(3) (*a*) Discuss the convergence of the simple iterative method.

(*b*) The equation $x^2 + ax + b = 0$ has two real roots α and β.
Show that the iteration method,

$$x_{k+1} = -\frac{(ax_k + b)}{x_k} \quad \text{is convergent near } x = \alpha \quad \text{if } |\alpha| > |\beta|$$

(c) The equation $x = f(x)$ is solved by the iteration method. $x_{k+1} = f(x_k)$ and a solution is wanted with a maximum error not greater than 0.5×10^{-4}. The first and second iterates were computed; $x_1 = 0.50000$ and $x_2 = 0.52661$. How many iterations must be performed further if it is known that $|f'(x)| \le 0.53$ for all values of x.

(4) (*a*) (i) What is the geometric interpretation of the Newton's formula for solving $f(x) = 0$.

(ii) With the usual notation prove that the condition for convergence of the Newton's method is $|f(x^*)f''(x^*)| < (f'(x^*))^2$; where x^* is the solution.

(*b*) Newton's method for solving the equation $f(x) = c$, where c is a real valued constant is applied to the function

$$f(x) = \begin{cases} \cos x & when |x| \le 1 \\ \cos x + (x^2 - 1)^2 & when |x| \ge 1 \end{cases}$$

For which c is $x_n = (-1)^n$; when $x_0 = 1$ and the calculation is carried out with no error?

(*c*) Discuss the advantages and disadvantages of using Newton's method.

(5) (*a*) With usual notation obtain the following.

(i) $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

(ii) $\Delta(\frac{1}{f_i}) = -\left(\frac{\Delta f_i}{f_i f_{i+1}}\right)$

(iii) $\Delta(\frac{f_i}{g_i}) = \left(\frac{(g_i \Delta f_i - f_i \Delta g_i)}{g_i g_{i+1}}\right)$

(iv) If $f(x) = e^{ax}$ Show that $\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$

(b) Complete the following difference table

x	0		5		10		15		20		25
y	...		...		...		...		...		1.7624
Δy		...		0.1243		...		...		...	
$\Delta^2 y$			0.0118		...		...		...		
$\Delta^3 y$				...		...		0.0115			
$\Delta^4 y$					0.0012		...				
$\Delta^5 y$						-0.0001					

(c) Using Newton's forward difference formula, find the interpolating polynomial which fits best for the given data.

(6) (a) Let $x_0, x_1, \ldots x_n$ be distinct numbers in the interval $[a,b]$ and $f \in c^{n+1}[a,b]$. Then prove that for each x in $[a,b]$, a number $\varepsilon(x) \in [a,b]$ exists such that

$f(x) - p_n(x) = \dfrac{f^{n+1}(\varepsilon(x))}{(n+1)!}\pi(x)$ where $p_n(x)$ is the Lagrange's interpolation polynomial of degree n and $\pi(x) = (x-x_0)(x-x_1)\ldots.(x-x_n)$

(b) In studies of radiation induced polymetrization, a source of gamma rays was employed to give measured doses of radiation. However the dosage varied with position in the apparatus, with these figures being recorded.

Positions in cm from base point	0	0.5	1.0	1.5	2.0	3.0	3.5	4.0
Dosage 10^5 radius/hr	1.90	2.39	2.71	2.98	3.20	3.20	2.98	2.74

For some reason the reading at 2.5cm was not reported, but the value of radiation there is needed.

Show how to fit an interpolating polynomials of degrees 2 and 3 to the data to supply the missing information.

(c) What do you think is the best estimate for the dosage level at 2.5cm?