The Open University of Sri Lanka
B.Sc Degree Programme – Level 04
Final Examination 2007/2008
Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

Duration :- Two and Half Hours.

Date :- 24-06-2008. **Time:- 10.00 a.m. – 12.30 p.m.**

Answer FOUR Questions only.

01. The retardation of a train with the power cut off is $\left(v^2+\frac{u^2}{4}\right)\text{ms}^{-2}$ where $v\ \text{ms}^{-1}$ is the speed and u is a constant. Initially $v = u$.

(a) Show that the speed will be halved in a distance $\frac{1}{2}\ln\left(\frac{5}{2}\right)$ metres in time $\frac{2}{u}\left[\arctan 2-\frac{\pi}{4}\right]$ seconds.

(b) Show also that the train will come to rest in a further distance $\frac{1}{2}\ln 2$ metres in additional time $\frac{\pi}{2u}$ seconds.

02. A particle of mass $4m$ is attached to the midpoint of a light spring of modulus $2mg$ whose ends are attached to two fixed points distant $8a$ apart in a vertical line. If the spring is of natural length $2a$, find the depth below the upper fixed point, A, of the position of equilibrium of the particle. When the particle is slightly disturbed from rest in a vertical direction show that it performs simple harmonic motion of periodic time $2\pi\sqrt{\left(\frac{a}{g}\right)}$.

03. Derive velocity and acceleration components in plane polar coordinates (r, θ).

A point starts from the origin in the direction of the initial line with velocity f/ω and with constant angular velocity ω about the origin and with constant negative radial acceleration $-f$. Show that the rate of growth of the radius velocity is never positive, but tends to the limit zero and prove that the equation of the path is $\omega^2 r = f(1-e^{-\theta})$.

04.(a) With usual notation derive the equation of the central orbit $\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2}$ where $u=\frac{1}{r}$ and P is the central force per unit mass.

(b) A particle moving with a central force $\frac{\mu}{r^3}$ per unit mass directed towards the pole is projected at a distance a with a velocity V perpendicular to the radius. Derive possible equations of the orbits.

05. Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}-\frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being condensed at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

A small body, of mass m_0, is projected vertically upwards in a cloud. Its initial speed is $(2gk)^{\frac{1}{2}}$. During its motion the body picks up moisture from the stationary cloud. Its mass at height x above the point of projection is $m_0(1+\alpha x)$, where α is a positive constant. Show that the greatest height h satisfies the equation $(1+\alpha h)^3=(1+3k\alpha)$.

06. The only force acting on a body, which is of mass M and is at a distance r from the centre of the Earth, is directed towards the centre of the Earth and is of magnitude $\frac{\mu M}{r^2}$, where μ is a constant. Show that the speed of a satellite of mass m moving in a circular orbit of radius a about the centre of the Earth is $\sqrt{\left(\frac{\mu}{a}\right)}$.

A second satellite, of mass $3m$, is moving in the same circular orbit as the first but in the opposite direction and the two satellite collide and coalesce to form a single composite body is governed by the two equations:

$$r^2\dot{\theta}=\sqrt{\left(\frac{a\mu}{4}\right)}$$

$$\ddot{r}=\frac{\mu(a-4r)}{4r^3}$$

where (r,θ) are the polar coordinates of the body with the centre of the Earth as pole. Find the values of r when $\dot{r}^2=0$.