THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
ADU5306/APU3150- Fluid Mechanics
FINAL EXAMINATION 2017/2018

Duration: Two Hours

Date: 30.03.2019 **Time: 09.30 a.m- 11.30 a.m**

Answer **FOUR** questions only. Standard notation are used throughout this paper.

1. The velocity vector in a flow field is given by

$$\mathbf{q} = \frac{k^2(-y\mathbf{i} + x\mathbf{j})}{x^2 + y^2} \quad (k = \text{constant}).$$

   (a) Show that the motion is a possible motion for an incompressible fluid.

   (b) Determine the equations of streamlines.

   (c) Show that the motion is of potential kind.

   (d) Determine the velocity potential.

   (e) Sketch the streamlines and equipotentials.

2. (a) In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Show that the components of velocity u, v, w are solutions of Laplace's equation.

(b) Find the vorticity vector of the fluid motion for the given velocity components: $v_r = \left(1 - \frac{A}{r^2}\right)\cos\theta$, $v_\theta = -\left(1 + \frac{A}{2r^3}\right)\sin\theta$, $v_\phi = 0$, where A is a constant.

(c) A velocity field is given by $\mathbf{q} = -x\mathbf{i} + (y+t)\mathbf{j}$. Find the stream function and the stream lines for this field at $t = 2$.

3. (a) Given Euler's equation of motion $\underline{\mathbf{F}} - \frac{1}{\rho} grad p = \frac{D\underline{\mathbf{q}}}{Dt}$ for a perfect fluid, show that it can be written in the form $\underline{\mathbf{F}} - \frac{1}{\rho} grad p = \frac{\partial \underline{\mathbf{q}}}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{\mathbf{q}} \times curl \underline{\mathbf{q}}$.

(b) Using the result in Part (a), derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

(c) Consider a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of $500\,mm^2$ and the exit has a bore area of $250\,mm^2$. Assuming there is no energy loss calculate the flow rate when the inlet pressure is $400\,Pa$.

4. A venturi meter is an instrument to measure the fluid velocity in pipes. The flow rate of a fluid in conduit and the discharge of a fluid flowing in a pipe may also be measured. The venturi meter is made up of a constant cross-section S_1 tapering to a section of smaller cross-section S_2 (also known as throat) and then gradually expanding to the original cross-section as shown in the Figure 1. A U-tube serving as a mercury manometer is attached to connect the board and narrow sections at A and B. Let q_1 and q_2 be the fluid velocities and p_1 and p_2 be the pressures at A and B respectively. Let ρ and σ be densities of the fluid and the mercury respectively.

(a) Find the velocity, q_1 of the fluid in terms of S_1, S_2, h, ρ and σ.

(b) Find the flow rate of the fluid flowing through the board section at A, in terms of S_1, S_2, h, ρ and σ.

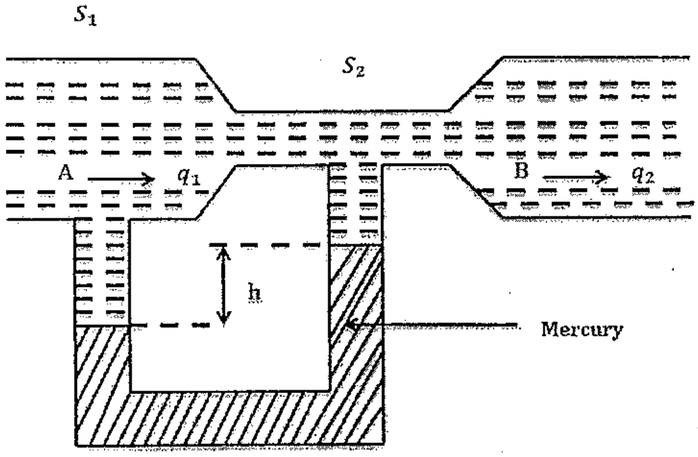

Figure 1: A veturi meter

5. (a) State the Navier-Stokes equation for a viscous incompressible fluid with constant viscosity.

   (b) Show that for an incompressible steady flow with constant viscosity, the velocity components

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1-\frac{y}{h}\right), \quad v = 0, \quad w = 0,$$

   where $h, U, dp/dx$ are constants and $p = p(x)$, satisfy the equation of motion, when the body force is neglected.

6. The complex potential of a fluid flow is given by $W(z) = 5\left(z + \frac{4}{z}\right)$.

   (a) Obtain the equation for the streamlines and velocity potential lines and represent them graphically.

   (b) Find the complex velocity at any point and determine its value far from the origin.

   (c) Find the stagnation points of the flow.