The Open University of Sri Lanka
B.Sc. Degree Programme
Final Examination 2007/2008
Pure Mathematics – Level 05
PMU3294/CSU3276 – Discreet Mathematics - Paper II

Duration: - Two & Half Hours.

Date: - 31-01-2008.   Time: - 1.30 p.m. – 4.00 p.m.

Answer Four Questions Only

1. There is a deck of 52 cards.
   (a) How many ways are there to select 2 cards without replacement from the deck ?

   (b) How many ways are there to select two cards from the deck such that
   (i) the first card is an Ace and the second card is a King?
   (ii) the first card is an Ace and the second card is not a King?
   (iii) the first card is an Ace and the second card is a Club?
   (iv) the first card is a Heart and the second card is a King?
   (v) the first card is a Heart and the second card is not a King?
   (vi) neither card is an Ace?
   (vii) at least one of the cards drawn is an Ace?

2. In a survey of students of a certain university the following information was obtained: 260 were taking statistics, 208 were taking mathematics, 160 were taking computer programming, 76 were taking statistics and mathematics, 48 were taking statistics and computer programming, 62 were taking mathematics and computer programming, 30 were taking mathematics, statistics and computer programming and 150 were taking none of these.
   (a) How many students were surveyed?
   (b) How many students were taking statistics and mathematics but not computer programming ?
   (c) How many were taking statistics and computer science but not mathematics?
   (d) How many were taking computer programming and mathematics but not statistics?
   (e) How many were taking statistics but not either mathematics or computer programming?
   (f) How many were taking mathematics but not either statistics or computer programming?
   (g) How many were taking computer programming but not either mathematics or statistics?

3. (a) Let (S, p) be a probability space and suppose that A is an event in (S, p).
Show that
(i) $p(A) = 1 - p(A^c)$; $A \in S$
(ii) If $A_1 \subseteq A_2$ then $p(A_1) \le p(A_2)$; $A_1, A_2 \in S$

(b) A bag contains 25 balls of which 18 are red and the rest are of other colors.
Three balls are drawn from the bag in succession. What is the probability that
all three balls are red?

(c) A letter is sent to 10 people in the first week of the year. The next week each
person who received a letter sends letters to 10 new people, and so on. How
many people have received letters
(i) after 10 weeks ?
(ii) at the end of the year?

4. (a) Draw a graph having the given properties or explain why such a graph
    does not exist.
    (i) Six vertices each of degree three
    (ii) Six vertices and four edges
    (iii) Simple graph of six vertices having degrees 1,2,3,4,5,5
    (iv) Tree with degrees 2,3,4,1,1,1 of 6 vertices.

(b) Find all simple cycles in the following graph.
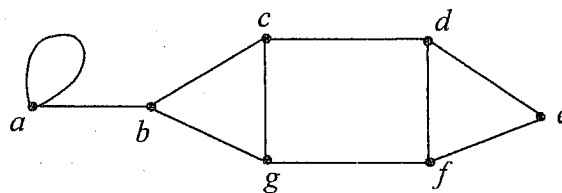
Find all simple paths from a to e of the above graph.

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5. Solve the following difference equations.

(i) $f(n) = f(n-1) + n, \qquad f(1) = 4$

(ii) $f(n) = 4f(n-1) + 5f(n-2), \qquad f(1) = 2, \; f(2) = 6$

(iii) $f(n+2) - 4f(n+1) + 13f(n) = 0$

6. (a) Show that the function $f(n)$ given by $f(n) = 1 - \dfrac{2}{n}$ is a solution of the first order difference equation $(n+1)f(n+1) + nf(n) = 2n - 3$.

(b) Find the general solution of each of the following difference equations.

(i) $f(n+2) - 2f(n+1) + f(n) = n^2 + n + 1$

(ii) $f(n+2) - 4f(n+1) - 5f(n) = n(2 + 5^n)$