100

The Open University of Sri Lanka
B.Sc. Degree Programme
Final Examination 2007/2008
Pure Mathematics – Level 05
PMU3294/CSU3276– Discreet Mathematics -Paper-I

Duration: - Two & Half Hours.

Date: - 31-01-2008. **Time: - 9.30 a.m. – 12.00 noon.**

Answer Four Questions Only

1. (a) Let p, q and r be the statements given below.

p: Today is Monday.
q: It is raining.
r: It is hot.

Write down each of the symbolic expressions in words.

(i) $(\sim p)\wedge(q\vee r)$

(ii) $\sim(p\vee q)\wedge r$

(iii) $(p\wedge q)\wedge\sim(r\vee q)$

(b) Suppose that the universe of discourse is the set of real numbers. Let $p(x,y)$ be the proposition '$x.y=4$' and let $q(x,y)$ be the proposition '$x>y$'. Indicate which of the following propositions are true and which are false. Justify your answer.

(i) $p(8,0.5)$

(ii) $\exists y\ p(2,y)$

(iii) $\forall x\exists y\ p(x,y)$

(iv) $\exists x\forall y\ p(x,y)$

(v) $\forall x\forall y\ [p(x,y)\Rightarrow q(x,y)]$

(vi) $\exists x\exists y\ [p(x,y)\wedge q(x,y)]$

2. (a) Prove that for any integer n, $n(n^2+5)$ is an integer multiple of 6.

(b) Prove by contradiction, that if x is a rational number and y is an irrational number then the sum $x+y$ is an irrational number.

3. Let X be the set of all four bit strings. (eg : 0011, 0101, 1000, ...) Define a relation R on X as $_{s_1}R_{s_2}$ if some substring of s_1 of length 2 is equal to some substring of s_2 of length 2.

(eg : $_{0111}R_{1010}$ because both 0111 and 1010 contain 01. Also $_{0111}\not{R}_{0001}$ because 1110 and 0001 does not share a common substring of length 2.)

Is this relation,
(i) reflexive?
(ii) symmetric?
(iii) antisymmetric?
(iv) transitive?
(v) a partial order?

4. Define an equivalence relation.

(a) Which of the following relations are,
(i) reflexive?
(ii) symmetric?
(iii) transitive?

$R' = \{(1,1),(2,2),(2,1),(1,2),(3,2)\}$
$R'' = \{(1,1),(2,2),(3,3),(2,1),(1,2)(3,2),(2,3)(1,3),(3,1)\}$
$R''' = \left\{(a,b) : a \equiv b \pmod{m} \quad a,b \in \mathbb{Z} \text{ and } m \in \mathbb{Z}^+\right\}$

Justify your answers.

(b) Let A be a non empty set and let $\sim$ be an equivalence relation on A. Show the following.
(i) $[a] \neq \phi$ for all $a \in A$.
(ii) $x \in [a] \Leftrightarrow [x] = [a]$ for all $a, x \in A$.
(iii) $[a] = [b] \Leftrightarrow a \sim b$ for all $a, b \in A$.
(iv) either $[a] = [b]$ or $[a] \cap [b] = \phi$ for all $a, b \in A$.

5. (a) Let the binary operation $*$ be defined on $A=\{a,b,c,d,e\}$ by means of the following operation table.

$*$	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

(ii) Is $*$ associative? Justify your answer.

(i) Compute $((a*c)*(e*d))*b$.

(iii) Is $*$ commutative? Justify your answer.

(b) Compute the following table so as to define a commutative binary operation $*$ on $S=\{a,b,c,d\}$.

$*$	a	b	c	d
a	a	b	c	
b	b	d		c
c	c	a	d	b
d	d			a

6. (i) Express $\{x\in\mathbb{R}: x>2 \Rightarrow x^2>9\}$ using intervals.

(ii) Prove that there is no real number x such that $x>0$ and for each $\varepsilon>0, x<\varepsilon$.

(iii) Give an example of a nonempty set X and a relation R on X such that R is symmetric and antisymmetric.

(iv) Let G be a group and let $a,b\in G$ such that $ab=ba$. Prove that $(ab)^n=a^n b^n$ for each positive integer n.