The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Final Examination 2008/2009
Pure Mathematics/Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics - Paper II

084

Duration : Two and Half (2 ½) Hours.

Date : 27-01-2009. **Time: 01.00 p.m. – 03.30 p.m.**

Answer Four Questions Only.

01. (i) Use the principle of induction to prove that, for each integer $n \geq 1$, a set with n elements has 2^n subsets.

(ii) Find the number of diagonals in a regular polygon with n sides. Deduce that the septagon is the only regular polygon with the number of diagonals is twice the number of sides.

(iii) Let n be a positive integer. Find the number of positive integers with n digits.

02.(i) Two regular dice are rolled and the values that show up are paired to form the outcome. Find

(a) the conditional probability that an odd number appears in at least one die given that at least one die shows 4.

(b) the conditional probability that at least one die shows 5 given that sum of the two values is divisible by 3.

(ii) Suppose B_1, B_2 are mutually disjoint events in a probability space (S, P) where $S = B_1 \cup B_2$. Let A be an arbitrary event in S.

Show that $P(B_i|A) = \dfrac{P(A|B_i)P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}$ for each $i = 1, 2$.

03.(i) Write down the set theoretic definition of the graph $G = G(V, E)$ whose adjacency matrix A is given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

Sketch a diagram of G.

(ii) Let n be a positive integer such that $n > 1$. Does there exist a graph G with adjacency matrix A such that the sum of the diagonal elements of A^2 is 7? Justify your answer.

(iii) Prove that if the vertices a and b of a graph G are joined by a path, then they are also joined by a simple path.

04. (i) Prove that a connected graph $G = G(V, E)$ is a tree if and only if $|E(G)| = |V(G)| - 1$.

(ii) Prove that if a forest F has c components, v vertices and e edges, then $v = c + e$.

(iii) Does there exist a non–degenerate tree with exactly one vertex of degree one? Justify your answer.

05. Solve the following difference equations.

(i) $f(n+1) = \frac{n}{2^n} f(n)$.

(ii) $f(n+1) - (n+1)f(n) = (n+1)3^n$.

(iii) $f(n+3) - 3f(n+2) - 10f(n+1) + 24f(n) = 0$.

06. Solve the following difference equations.

(i) $f(n+2) - 6f(n+1) + 18f(n) = 0$.

(ii) $f(n+2) - 5f(n+1) + 6f(n) = 2^{n+1} + 3^{n+1}$.

(iii) $f(n+2) - 10f(n+1) + 25f(n) = 0$ given that $f(1) = 5$ and $f(2) = 100$.