The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Final Examination 2008/2009
Pure Mathematics/Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics - Paper I

082

Duration : Two and Half (2 ½) Hours.

Date : 27-01-2009. **Time: 9.30 a.m. – 12.00 noon.**

Answer Four Questions Only.

01. Find the truth value of each of the following statements. Justify your answer.

(i) Every real number is rational, or every real number is irrational.

(ii) There exists a positive integer n such that n is even and n is prime.

(iii) If x is a real number such that $x^2 > 1$, then x is a real number such that $x > 1$.

(iv) There exists a prime number p such that p^2 is prime, if and only if, there exists a rational number r such that r^2 is irrational.

02.(i) Let a_1, a_2, a_3, a_4 and a_5 be real numbers such that $a_1 + a_2 + a_3 + a_4 + a_5 = 5$. Prove that there exists a $k \in \{1, 2, 3, 4, 5\}$ such that $a_k \geq 1$.

(ii) Let $A = \{1, 2, 3, 4\}$. Prove or disprove that, for each $a \in A$, $a^2 + a + 5$ is a prime number. Name the method of your proof.

(iii) Let $a_1, a_2, a_3, \ldots, a_n, \ldots,$ be a sequence of real numbers which satisfies the recurrence relation, $(n+2)a_{n+1} - (n+1)^2 a_n = -n^3 + n + 1$, for each positive integer n. It is given that $a_1 = 1$. Prove that $a_n = \dfrac{n^2+1}{n+1}$ for each $n \geq 1$.

03. (i) Write the negation of each of the following statements without using the words "no" and "not".

(a) $a_n > 0$ for all n.

(b) For each $x \in \mathbb{R}$, for each $y \in \mathbb{R}$, if $x < y$ then $f(x) < f(y)$.

(c) At least one student scored more than 70 points in the final exam.

(d) $I \in \mathbb{B}$ if and only if for each $a \in I$, there exists $b \in I$ such that $b > a$.

(e) If $n \in \mathbb{N}$ then $6n + 1 \in \mathbb{P}$ or $6n - 1 \in \mathbb{P}$.

(ii) Write the following statements using words without using the symbols ∀ and ∃.

(a) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n = 1$.

(b) $\exists n \in \mathbb{Z}, \forall x \in \mathbb{R}, x + n = x$.

(c) $\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \exists x \in \mathbb{Q}, \frac{x}{x+1} = \frac{m}{n}$.

04. Show that the set $G = \left\{x, -x, \frac{1}{x}, -\frac{1}{x}\right\}$ of functions under the binary operation $\circ$ of composition of functions forms a group.

Is $(G, \circ)$ a commutative group? Justify your answer.

List all the subgroups of $(G, \circ)$.

Is $(G, \circ)$ a cyclic group? Justify your answer.

Let H be the group of the set of integers modulo 5 excluding 0 under multiplication. Is H isomorphic to G? Justify your answer.

05. Let G be a group with at least four distinct elements and let $\mathscr{A} = \{H : H \text{ is a subgroup of } G\}$. Define the relation R on $\mathscr{A}$ as, for each $H_1, H_2 \in \mathscr{A}$, $H_1 R H_2$ if H_1 is a subgroup of H_2.

(i) Prove that R is a partial order.

(ii) Show that R is not an equivalence relation.

(iii) Is R a total order for each G? Justify your answer.

06. Let X be the set of nonzero real numbers.

(i) Define the relation R_1 on X as, for each $a, b \in X$, $a \; R_1 \; b$ if $\frac{a}{b}$ is a rational number. Prove that R_1 is an equivalence relation on X.

(ii) Define the relation R_2 on X as, for each $a, b \in X$, $a \; R_2 \; b$ if $\frac{a}{b}$ is an irrational number.

Show that R_2 is symmetric, non–reflexive and non–transitive.