THE OPEN UNIVERSITY OF SRI LANKA
B.SC. DEGREE PROGRAMME – LEVEL 05
FINAL EXAMINATION-2008/2009
PHU 3148/PHE 5148 MATHEMATICAL PHYSICS
DURATION: TWO AND HALF (2 $^1/_2$) HOURS

Date: 18th July 2009 **Time: 1.30 p.m. to 4.00 p.m.**

Answer **Four (4)** questions only

(1) (a) Calculate the force on the proton, in Newtons in a magnetic field of intensity (B) 0.01 Tesla directed along z- axis, when the proton moves across the magnetic field with velocity 2.6×10^5 m s^{-1} at an angle of 45^0 with the x-axis on the x-z plane.

Electrical charge of the proton (q) $= 1.6 \times 10^{-19}$ C,
Force on the proton due to magnetic field = q (**V** × **B**).

(b) (i) If $\mathbf{A} = x^2\,\mathbf{i} - 2z^2y^2\mathbf{j} + xyz\mathbf{k}$ find $\nabla.\mathbf{A}$ at point (2,1,-1)
(ii) If $\phi = xyz$ find $\mathbf{A} \times (\nabla\phi)$

(c) A rigid body is rotating with angular velocity ω about a vertical axis. Let **r** be the position vector of any point P of the body.
If the linear velocity at point $\mathbf{r} = x\,\mathbf{i} + y\mathbf{j} + z\,\mathbf{k}$ is defined as **V**= ω × **r**, find the curl of the linear velocity at the above point and hence show that it is independent of the location **r**.

(2) (a) Solve the following linear equations by the matrix method.

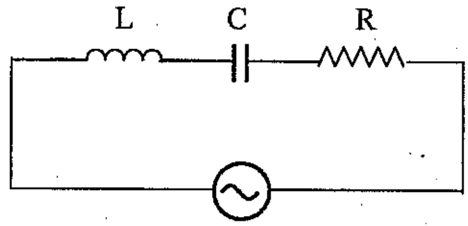
$$\begin{aligned} x \quad + 2z &= 1 \\ 3x - y + z &= 2 \\ 4y + 5z &= -1 \end{aligned}$$

(b) Find the eigen values and the eigen vectors of the matrix.

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$$

(3) An electrical circuit consisting of an inductance(L), a capacitance (C) and a resistance (R) is connected to an voltage source ; $V_o \sin \omega t$. If the current I is flowing at any instant, the voltage equation is

$$\frac{q}{C} + IR + L\frac{dI}{dt} = V_o \sin \omega t$$



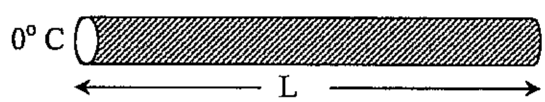
The rate of change of electrical charge at any point of the circuit is defined as current; $I = \frac{dq}{dt}$

(a) Solve the differential equation for the charge q and then show that

$$I = \frac{V_0}{\sqrt{[R^2 + (L\omega - \frac{1}{C\omega})^2]}} \cos(\omega t - \theta) \quad \text{where } \tan\theta = \frac{\omega R / L}{(1/LC - \omega^2)}$$

(b) The dominator of the above solution is known as the impedance in the circuit . Find the frequency of the external voltage source to resonate the circuit.

(4) A cylindrical metal rod which is initially at the temperature of T_0 is completely covered with a insulator except one flat surface as shown in the following figure.



The temperature; $T(x,t)$ of the bar is modeled by the diffusion equation ;

$$\frac{\partial T(x,t)}{\partial t} = k\frac{\partial^2 T(x,t)}{\partial x^2} \quad (0 < x < L \;\&\; t > 0)$$

(a) Write down the initial condition, and the boundary conditions for T at the two ends of the rod at any time.

(b) Solve the differential equation and determine the expression for $T(x,t)$

(5) The electronic circuits of an electric musical instrument can originally generate sinusoidal voltage signals with different frequencies and different amplitudes. It produces any musical sound by composing the original sinusoidal signals.

(a) Find the series of sinusoidal voltage signals using the Fourier series method to compose square wave voltage signal $V(t)$;

$$V(t) = 4 \text{ volts} \quad \text{for } 0 < t < \pi$$
$$= -4 \text{ volts} \quad \text{for } \pi < t < 2\pi$$

(b) Draw a graph using the first three terms of the above series to compare the real square wave signal and the composed three term approximation.

(6) (a) Define the Laplace transformation.

(b) If $f(t)$ is continuous for $t > 0$ and exponential order as $t \rightarrow \infty$, while $f'(t)$ is sectionally continuous for $t > 0$ and if $L\{f(t)\} = F(s)$

(i) Show that $L\{f'(t)\} = sF(s) - f(0)$

(ii) Hence find $L\{f''(t)\}$

(c) Solve the following differential equation for $f(t)$ using the Laplace transformation.

$$f''(t) + 8f'(t) + 25f(t) = 150$$

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