The Open University of Sri Lanka
B.Sc Degree Programme
Final Examination-2008/2009
Applied Mathematics-Level 05
AMU 3189/AME 5189 – STATISTICS II

Duration:- Two & Half Hours

Date:-30.06.2009 **Time:-10.00a.m.-12.30p.m.**

Answer Four Questions Only

01). A consumer organization wishes to study about the mean lifetime (hours of writing) of an inexpensive brand of ballpoint pens. The organization purchased 40 such pens and inserted each in a specially constructed machine that cause the pen to write continuously until the ink runs out. The following table gives the observed life times. Assume that the life times are normally distributed with unknown population mean μ and population variance σ^2.

31.7	26.2	35.0	34.8	33.1	30.8	34.1	34.4	29.5	31.4
29.0	29.7	32.5	25.3	30.6	37.7	28.7	23.1	26.7	25.7
31.0	28.7	30.7	36.4	27.3	32.1	29.2	30.1	25.9	26.3
27.5	28.6	37.2	31.3	22.7	27.4	31.7	30.0	29.6	33.6

i). Derive moments estimators and maximum likelihood estimators for μ and σ^2.

ii). Estimate parameters separately by the method of moments and maximum likelihood.

iii). Which estimator is an unbiased estimator for σ^2? Give reasons for your answer.

02). A dietitian hopes to reduce a person's cholesterol level by using a special diet supplemented with a combination of vitamin pills. Five subjects were pre–tested and then placed on diet for two weeks. Their cholesterol levels were checked after a two–weeks period. The results are shown in the following table. Cholesterol levels are measured in milligrams per deciliter.

Subject	1	2	3	4	5
Before (X_1)	192	210	206	200	192
After (X_2)	172	181	198	195	193

Assume $X_1 \sim N(\mu_1, \sigma_1^2)$ **and** $X_2 \sim N(\mu_2, \sigma_2^2)$.

i). Construct a 95% confidence interval for $\frac{\sigma_1}{\sigma_2}$.

ii). What is the length of the confidence interval you constructed in part (a) above?

iii). In relation to this study, interpret the confidence interval you constructed in part (a) above?

03). a). Let X be a random variable that takes the values 1 and 2 with probabilities θ and $(1-\theta)$ respectively. Compute

i). mean of X

ii). Variance of X

b). Let $X_1, X_2, ..., X_{10}$ be a random sample of 10 values taken by the random variable X, described in part (a). Compute the biases and mean squared errors of each of the following estimators for θ.

i). X_1

ii). $\frac{X_1 + X_2 + ... + X_{10}}{10}$

iii). $2 - X_2$

c). Out of the three estimators, which estimator do you recommend as an estimator for θ ? Give reasons for your answer.

4).a). A manufacturer of sprinkler systems used for fire protection in office buildings claims that the mean activation temperature of his product is 130 ^{0}F. A sample of 9 systems was tested. The sample average and sample standard deviation of activation temperatures were 131.08 ^{0}F and 1.5 ^{0}F respectively. Assume that the distribution of activation times is normally distributed.

i). Write down the hypotheses appropriate to test the manufacturer's claim.

ii). Does the data contradict the manufacturer's claim? Briefly explain your answer. Use a 5% significance level.

b). The heights (in cm) of one–month old plants of a plant species A is found to be normally distributed with mean μ and variance 4. Let $X_1, X_2, \ldots, X_n$ denote the heights of n randomly chosen one–month old plants of species A.

i). Using Neyman Pearson Lemma derive a test for testing H_0 : $\mu=1$ vs H_1: $\mu=3$ at 5% significance level.

05). A manufacturing process is regarded as satisfactory if the percentage of defectives does not exceed 5%. To test the condition of the process, the quality control manager examines a random sample of twenty items from the output of each day. If the number of defectives in the sample exceeds 2, the process is labeled as unsatisfactory. Otherwise, the process is labeled as satisfactory.

i). Explain the following terms in relation to the above experiment.

a. Type II error

b. Size of the test

c. Power of the test

ii). Compute the size of the test.

iii). Obtain the power function of the test and sketch it.

iv). If the actual percentage of defectives is 10%, what is the probability that the test will detect the process as unsatisfactory?

06. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the Poisson distribution with mass function given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \; x = 0, 1, 2, \ldots$$

i) Find the Cramer Rao lower bound for the variance of an unbiased estimator for λ.

ii) Show that $\frac{\sum x_i}{n}$ is a minimum variance unbiased estimator for λ.

iii) Let $X_1, X_2, \ldots, X_9$ be a random sample of 9 observations from the above density. State whether each of the following is a minimum variance unbiased estimator for λ or not. In each case, give reasons for your answer.

a) $\frac{X_1 + X_2}{2}$

b) $\frac{X_1 + X_2 + X_3 + X_4}{6}$

c) $\frac{X_1 + X_2 + X_3}{9}$

iv) Number of telephone calls received by an operator during a 10–minute period is known to follow a Poisson distribution with parameter λ. Actual number of calls received by the operator on 9 days were $\{4, 0, 2, 6, 5, 4, 3, 2, 4\}$.

a) Obtain an estimate for λ using a minimum variance unbiased estimator.

b) Does the estimator you used have the least variance compared to any estimator for λ? Give reasons for your answer.