The Open University of Sri Lanka
B.Sc Degree Programme
Final Examination -2008/2009
Applied Mathematics - Level 5
AMU 3181/AME 5181-Fluid Mechanics

Duration : Two and half hours

Date: 22.07.2009 **Time: 10.00am-12.30pm**

Answer FOUR Questions Only.

01. Two stream functions of two-dimensional fluid motions are given by

(a) $\psi_1 = Ur\sin\theta$ (b) $\psi_2 = \frac{-\mu}{r}\sin\theta$

where U and μ are positive constants and (r, θ) denote plane polar co-ordinates.

(i) Find the velocity components (u_r, u_θ) and sketch the stream lines, indicating the direction of flow, separately, for each stream function.

(ii) In the combined motion with the stream function ψ given by $\psi = \psi_1 + \psi_2$, show that the circle $r = a = \sqrt{\frac{\mu}{U}}$ may be replaced by a rigid boundary.
By eliminating μ, re-write ψ as a function of (r, θ) and the constants U and a.

Identify the stagnation points of this motion.

(iii) Let $\underline{V} = \left(kx^2y + \frac{\alpha}{y}\right)\underline{i} + \left(-kxy^2 + \frac{\beta}{x}\right)\underline{j} + \varepsilon\underline{k}$ be the velocity field of a flow and $\rho = \rho_0 \exp\{-\gamma(xy - ct)\}$ be the density of the fluid, where $k, \alpha, \beta, \varepsilon, \gamma, \rho_0$ and c are constants.

(a) Is this an incompressible flow? Give reasons for your answer.

(b) Does it involve an irrotational flow? Give reasons for your answer.

02. (i) Obtain the relationship between the velocity potential ϕ and the stream function ψ representing the same fluid motion, stating the conditions under which they exist.

(ii) Verify that the velocity potential $\phi = -m\log\left(\frac{r_1}{r_2}\right)$, where $r_1^2 = (x-a)^2 + y^2$ and $r_2^2 = (x+a)^2 + y^2$, m and a being constants, represents a possible motion.

(iii) Find the components of the velocity and show that the fluid speed is $u = \frac{2am}{r_1 r_2}$.

Show also that the stream function may be expressed in the form,

$\psi = -m\tan^{-1}\left(\frac{2ay}{x^2+y^2-a^2}\right)$.

03. (i) A wine glass has the form of a right circular cone of base radius a and depth h. The glass is half full of water. What is the geometrical shape of the free surface of water when the whole system is rotated about the z–axis in the vertical direction. Show that the water will just reach to the brim of the glass if the whole system is made to rotate about the same axis with angular velocity ω given by $\omega^2 = \frac{2gh}{3a^2}$.

(ii) If ω is gradually increased then show that the water will remain in contact with the brim if $\omega^2 = \frac{gh}{a^2}$ and then there is only half of the original quantity of water left in the glass.

(iii) What happen when ω is increased still further?

04.(i) A flow is two dimensional between two parallel plates which are situated horizontally in the x direction. The velocity u_1 is a function of z and t only. Assuming that pressure does not change in the y direction, show that the equation of motion of the system along the x direction is given $\rho\frac{\partial u_1}{\partial t} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u_1}{\partial z^2}$, where ρ is the density, μ is the coefficient of viscosity and P is the pressure.

(ii) Further, consider a two dimensional steady flow in the x - direction, where the plates are parallel to (x, y) plane and are moving with the velocities u and v at $z = 0$ and $z = h$ respectively. If the pressure gradient along the x – direction is a constant c, show that the velocity profile is given by $u_1 = \frac{c}{2\mu}(z^2 - hz) + \frac{z}{h}(v-u) + u$.

05. (i) Assuming the Euler's equation in the form of $\frac{\partial U}{\partial t}+\nabla\left(\frac{1}{2}U^2\right)-U\wedge(\nabla\wedge U)=F-\frac{1}{\rho}\nabla P$, in the usual notation, show that $\frac{P}{\rho}+\frac{1}{2}U^2-\Omega=Constant$, along the stream lines.

(ii) A liquid of density ρ is flowing along a horizontal pipe of variable cross- section and the pipe is connected to differential pressure gauges at two points A and B. Show that the mass m of the liquid flowing through the pipe per second is given by

$$m=\sigma_1\sigma_2\sqrt{\frac{2\rho(P_1-P_2)}{(\sigma_1^2-\sigma_2^2)}} \text{ provided } (P_1>P_2).$$

Here σ_1 and σ_2 $(\neq\sigma_1)$ are the areas of the cross-sections of the pipe and P_1 and P_2 are pressures at A and B respectively.

(iii) Suppose the amount of liquid flow per second is put into a cylindrical glass of constant diameter a. Find the height of the liquid in the glass.

06. (i) Define each of the following terms:

(a) Viscosity

(b) Non-uniform flow

(c) Steady flow

(d) Ideal flow

(e) Incompressible and Irrotational flow

(ii) Show that the centre of pressure is independent of the orientation of a submerge object in a liquid.

(iii) Show by the method of dimensional analysis, the pressure rise ΔP generated by a pump that depends on the impeller diameter D, its rotational speed N, the fluid density ρ and viscosity μ and the rate of discharge $\dot{V}$,may be expressed as

$$\Delta P=\rho N^2D^2\,\phi\left[\left(\frac{\dot{V}}{ND^3}\right),\left(\frac{\rho ND^2}{\mu}\right)\right], \text{ where } \phi(\Delta P,D,N,\rho,\mu,\dot{V})=0.$$