The Open University of Sri Lanka
B.Sc Degree Programme/Continuing Education Programme
Final Examination 2008/2009
AMU 1181/AME 3181 – Differential Equations
Level 03 - Applied Mathematics

**Duration :- 2 Hours.**

**Date :- 19-01-2009.** **Time:- 1.30 p.m. – 03.30 p.m.**

**Answer FOUR questions only.**

01. (a) The general solution of the differential equation $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ depends on the nature of the solutions of its characteristics equation, where $a$, $b$ and $c$ are constants and $a \neq 0$. Write the type of solutions of the auxiliary equation and the corresponding general solutions of the given differential equation.

(b) Show that any constant–coefficient second order differential equation $ay''+by'+c=0$ whose characteristic equation has a double root $r$ must have the form $y''-2ry'+r^2y=0$.

Let $y_1(t)=u(t)e^{rt}$, where $u(t)$ is a twice differentiable function of $t$.

Find $y_1'$ and $y_1''$ and simplify $y_1''-2ry_1'+r^2y_1$.

Find $u(t)$ if $y=y_1(t)$ is a solution of the equation $y''-2ry'+r^2y=0$.

02. Solve each of the following differential equations.

(a) $\left(x^2-3y^2\right)dx+2xydy=0$

(b) $\frac{dr}{d\theta}+r\tan\theta=\cos^2\theta$

(c) $\frac{dy}{dx}-\frac{y}{x}=-\frac{y^2}{x}$

(d) $\frac{dy}{dx}=\frac{x^2y^2}{1+x}$.

03.(a) Define a "Recurrence Relation" and its "order".

(b) A person takes a loan of Rs. 50,000/= from a bank at an annual interest rate 10% compounded monthly. The loan must be repaid in 2 years by making 24 equal monthly installments. Find the monthly installment.

(c) Solve the recurrence relation

$$T(n)=\begin{cases}3 & ;n=0\\ 17 & ;n=1\\ 10T(n-1)-25T(n-2) & ;n>1\end{cases}$$

04. (a) Formulate Euler's method for solving the equation $\frac{d^2y}{dx^2}+y=0$.

(b) Apply Euler's method to find $y(1)$ using a step length $h$ = 0.2 for the differential equation $\frac{d^2y}{dx^2}+y=0$ with the initial conditions $y=0$ and $\frac{dy}{dx}=1$ when $x$ = 0.

(c) Solve the differential equation in part (b) and find the absolute error.

05. (a) A spring of length 25cm is stretched to a length of 30cm by the weight of an object of mass 100 grams. Determine the spring constant.

(b) Find the circular frequency and period for the spring mass system in part (a).

(c) Suppose the mass is pulled down 1cm and released with an upward velocity of 4cm/s. Determine the amplitude of the subsequent motion.

(d) Verify that $y=c_1\cos 3t+c_2\sin 3t$ satisfies the differential equation $y''+9y=0$ regardless of the values of $c_1$ and $c_2$.

(e) Using part (d), determine the solution obtained in part (c) in amplitude–phase shift form and graph the solution.

06. (a) Show that cos $2t$ and sin $2t$ are solutions of the differential equation $y''+4y=0$.

(b) Using part (a) find the general solution of the non–homogenous differential equation $y''+4y=\cos 5t$.

(c) If $y(0)$ = 0 and $y'(0)=0$ show that the particular solution of $y''+4y=\cos 5t$ is $y=\frac{2}{21}\sin\frac{7}{2}t\sin\frac{3}{2}t$.