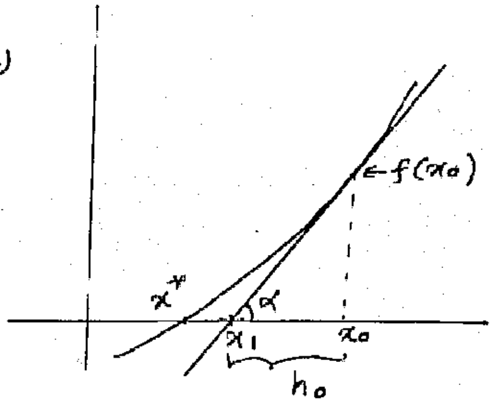
Closed Book Test - 2010/2011

Answer Guide - AMU 2185/AME4185

1. (a)



$x_1 = x_0 - h_0$.

$\tan\alpha = \frac{f(x_0)}{h_0} = f'(x_0)$

$\therefore h_0 = f(x_0) / f'(x_0)$

$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)} \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

(b) $f(x) = x^3 - 788$

$f'(x) = 3x^2$

$x_{n+1} = x_n - \frac{(x_n^3 - 788)}{3x_n^2}$; $x_0 = 7.0$.

$x_1 = 7 - \frac{(7^3 - 788)}{3\times 7^2} = 10.02721$

$x_2 = 9.29724$

$x_3 = 9.23693$

$x_4 = 9.23653$ & $x_5 = 9.23653$.

$\therefore$ root is 9.2365.

2 (a) Refer to page nos 80 and 81.

(b) $f(x) = x^3 + x^2 + 12x - 24 = 0$

$f(0) = -24$, $f(1) = -10$, $f(2) = 12$

$\because f(1) \cdot f(2) < 0$ roots lies between 1 & 2.

$f'(x) = 3x^2 + 2x + 12$

$x_{n+1} = x_n - \frac{(x_n^3 + x_n^2 + 12x_n - 24)}{(3x_n^2 + 2x_n + 12)}$

$x_0 = 1.5$

$x_1 = 1.5172$

$x_2 = 1.5172$

$\therefore$ root is 1.5172

1	1	12	−24	1·5172
	1·5172	3·8191	24·0007	
1	1·5172	15·8191	0·0007	

$q(x) = x^2 + 2\cdot5172x + 15\cdot8191$

$$x = \frac{-2\cdot5172 \pm \sqrt{(2\cdot5172)^2 - 4\times1\times15\cdot8191}}{2\times1}$$

$= -1\cdot2586 \pm 3\cdot7730i$

$\therefore$ roots are $1\cdot5172$, $-1\cdot2586 \pm 3\cdot7730i$.

(3) (a) The polynomial curve of degree n through the $(n+1)$ points (x_i, y_i) $i = 0,1,..n$ is $y = p(x)$ with,

$$p(x) = \sum_{i=0}^{n} L_i(x)\, y_i \quad \text{, where}$$

$$L_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

(i) $p(x)$ is a polynomial of degree n (ii) $L_i(x_i) = 1$ (iii) $L_i(x_j) = 0$ if $j \neq i$.

(b) $$p(x) = \frac{(x-2\cdot5)(x-3)}{(2-2\cdot5)(2-3)} \times 0\cdot69315 + \frac{(x-2)(x-3)}{(2\cdot5-3)(2\cdot5-3)} \times 0\cdot91629 + \frac{(x-2)(x-2\cdot5)}{(3-2)(3-2\cdot5)} \times 1\cdot0986$$

$= -0\cdot08164x^2 + 0\cdot81366x - 0\cdot60761$

$\ln(2\cdot3) \approx p(2\cdot3)$

$= -0\cdot08164\times(2\cdot3)^2 + 0\cdot81366(2\cdot3) - 0\cdot60761$

$= 0\cdot8319$.

$$E(x) = \frac{\phi(x)\, f^{n+1}(\xi)}{(n+1)!} \quad ; \quad \phi(x) = (x-2)(x-2\cdot5)(x-3)$$

$$= \frac{\phi(x)}{3!}\left[\frac{d^3}{dx}(\ln x)\right]_{x=c} = \frac{(x-2)(x-2\cdot5)(x-3)}{3!}\cdot\frac{2}{c^3} \quad ; \quad 2 < c < 3$$

$$|E(2\cdot3)| = \left|\frac{(2\cdot3-2)(2\cdot3-2\cdot5)(2\cdot3-3)}{6}\right| \times \left|\frac{2}{c^3}\right|$$

$\leq 0\cdot007 \times 0\cdot25 = 0\cdot00175 \leq \frac{1}{2}\times10^{-2}$

$\left[\because \max\left|\frac{2}{c^3}\right| = \frac{2}{8} = 0\cdot25\right]$

$\therefore \ln(2\cdot3) = 0\cdot83$.