The Open University of Sri Lanka

B.Sc Degree Programme/ Continuing Education Programme

Open Book Test (OBT)- 2010/2011

Level 04- Pure Mathematics

PMU 2191/PME 4191 – Vector Analysis

Duration :- One and half hours

Date:- 08.09.2010 **Time:- 4.00p.m.-5.30p.m.**

Answer All Questions.

1. (a) Consider the equation $\left(p+\frac{a}{v^2}\right)(v-b)=ct$, where the variables p, v and t denote pressure, volume and temperature respectively, and a, b, c are constants.

Find $\frac{\partial p}{\partial v}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial t}{\partial p}$ and show that $\left(\frac{\partial p}{\partial v}\right)\left(\frac{\partial v}{\partial t}\right)\left(\frac{\partial t}{\partial p}\right)=-1$.

(b) Considering the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ show that $xz\frac{\partial^2 z}{\partial x^2}+x\left(\frac{\partial z}{\partial x}\right)^2-z\left(\frac{\partial z}{\partial x}\right)=0$.

(c) Considering suitable multivariable function estimate the value of $2.36^2\sqrt{5.68^2+4.10^3}$.

2. (a) Find the stationary points of the function $f(x,y)=3x^3-9xy^2-2y^3-36y^2+96y$, and determine their nature.

(b) Find the second order Taylor polynomial for the function $z(x,y)=\sqrt{2x+5y}$ about the point $(3, 2)$.

3. (a) Find a unit vector normal to the surface $x = y^2 + z^2$ at the point $(2, 1, -1)$. Is there any other unit normal vector?

(b) A scalar field is given by $f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$, where a, b and c are constants. Find the directional derivative of $f(x, y, z)$ at the point (x, y, z) in the direction of the vector $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$.

(c) Evaluate the scalar line integral of $\underline{F} = y\underline{i} + x\underline{j} + z^2\underline{k}$ along the path C, where C is the part of the helix with parametric equations $x = \cos t$, $y = \sin t$, $z = t$ and $0 \leq t \leq 2\pi$.