

THE OPEN UNIVERSITY OF SRI LANKA  
B.Sc. DEGREE PROGRAMME - LEVEL 04  
OPEN BOOK TEST - 2010/2011  
PURE MATHEMATICS  
PUU2142 - Linear Algebra



DURATION: ONE AND HALF (1½) HOURS

Date : 31. 08. 2010

Time: 4.00 pm. To 5.30 pm

ANSWER ALL QUESTIONS.

1. (a) Solve the following system of equations using Gaussian elimination.

$$\begin{aligned}x_1 + 2x_2 + x_3 + x_4 &= 4 \\2x_1 + 4x_2 - x_3 + 2x_4 &= 11 \\x_1 + x_2 + 2x_3 + 3x_4 &= 1.\end{aligned}$$

(b) Check whether the following matrices are orthogonal or not.

$$(i) \begin{pmatrix} 5 & -12 \\ 13 & 13 \\ 12 & 5 \\ 13 & 13 \end{pmatrix} \quad (ii) \begin{pmatrix} 1 & 3 \\ \sqrt{10} & \sqrt{10} \\ -3 & 1 \\ \sqrt{10} & \sqrt{10} \end{pmatrix} \quad (iii) \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix}.$$

2. (a) Find the echelon form of the following matrices.

$$(i) A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{pmatrix} \quad (ii) \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}.$$

(b) Find the inverse of the matrix  $A$  where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

3. (a) Find the rank of the following matrix.

$$\begin{pmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) Use Cramer's rule (if applicable) to find the solutions of following systems of linear equations.

(i)  $3x + y = 1$   
 $5x + 2y = 3$

(ii)  $x + 2y + 3z = 3$   
 $2x - z = 4$   
 $4x + 2y + 2z = 5$