ANSWER ALL QUESTIONS.

1. (a) Solve the following system of equations using Gaussian elimination.

$$x_1 + 2x_2 + x_3 + x_4 = 4$$
$$2x_1 + 4x_2 - x_3 + 2x_4 = 11$$
$$x_1 + x_2 + 2x_3 + 3x_4 = 1.$$

(b) Check whether the following matrices are orthogonal or not.

(i) $\begin{pmatrix} \dfrac{5}{13} & \dfrac{-12}{13} \\ \dfrac{12}{13} & \dfrac{5}{13} \end{pmatrix}$
(ii) $\begin{pmatrix} \dfrac{1}{\sqrt{10}} & \dfrac{3}{\sqrt{10}} \\ \dfrac{-3}{\sqrt{10}} & \dfrac{1}{\sqrt{10}} \end{pmatrix}$
(iii) $\begin{pmatrix} \dfrac{1}{3} & \dfrac{2}{3} & \dfrac{2}{3} \\ \dfrac{2}{3} & \dfrac{1}{3} & \dfrac{-2}{3} \\ \dfrac{2}{3} & \dfrac{-2}{3} & \dfrac{1}{3} \end{pmatrix}.$

2. (a) Find the echelon form of the following matrices.

(i) $A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{pmatrix}$
(ii) $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}.$

(b) Find the inverse of the matrix A where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

3. (a) Find the rank of the following matrix.

$$\begin{pmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) Use Cramer's rule (if applicable) to find the solutions of following systems of linear equations.

(i) $3x + y = 1$
    $5x + 2y = 3$

(ii) $x + 2y + 3z = 3$
    $2x \quad\quad - z = 4$
    $4x + 2y + 2z = 5$