The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Closed Book Test (CBT) - 2010/2011
Pure Mathematics / Computer Science
PMU 3294/ CSU 3276/ PME 5294 – Discrete Mathematics

Duration: - One & Half Hours

Date: - 21-10-2010. **Time: - 4.00 p.m. – 5.30 p.m.**

Answer All Questions.

1. (a) Let the number plate of a vehicle consists of four different letters followed by four digits. Find

    (i) the total number of number plates that can be made,
    (ii) the number of number plates in which the first digit cannot be 0,
    (iii) the number of number plates in which no digit repeats.

   (b) In how many ways can seven boys and five girls stand in a line so that no girls are next to each other.

   (c) Find the number of ways in which five Mathematics books, four Chemistry books and three Physics books can be placed on a shelf if

    (i) there are no any restrictions,
    (ii) all books of the same subject are together.

   (d) In how many ways can the fifteen students take three different tests if five students are to take each test.

2. (a) Cricket teams Australia and England play for Ashes cup. But the probability of the team Australia wins a match is $0 \cdot 6$. If the tournament is scheduled so that the team that **first** wins two of three matches wins the Ashes cup and the tournament is over.

    (i) Draw a tree diagram to find the number of possible ways in which the tournament can occur,
    (ii) Find the probability that the team England wins the Ashes cup.

(b) Let A and B be two events with $P(B) > 0$. Define $P(A/B)$, the conditional probability of A given B. Find $P(A/B)$ if

(i) B is a subset of A,
(ii) A and B are mutually exclusive,
(iii) A and B are independent.

(c) In a survey of 100 students 55 like cricket, 38 like football and 23 like both cricket and football. What is the probability that a student randomly selected from this group likes exactly one of cricket or football.

3. (a) What is the largest possible number of vertices in a graph with 30 edges if all the vertices have degree at least 3.

(b) Let G be a graph with set of four vertices $\{v_1, v_2, v_3, v_4\}$, whose adjacency matrix A is given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

(i) Determine the number of paths of length three joining v_2 and v_4, What are those paths?
(ii) Deduce that G is connected,
(iii) Is G a tree? Justify your answer,
(iv) The subgraph H of G is defined by $V(H) = \{v_1, v_3, v_4\}$ and $E(H) = \{\{v_1 v_3\}, \{v_1 v_4\}, \{v_3 v_4\}\}$, Determine whether H is a component of G. Justify your answer.

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Model Answer

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01. (a) □□□□ □□□□

Different letters — digits can repeat

(i) $26\times25\times24\times23\times10\times10\times10\times10$

(ii) $26\times25\times24\times23\times9\times10\times10\times10$

(iii) $26\times25\times24\times23\times10\times9\times8\times7$

(b) Let A, B, C, D, E, F, G are 7 boys.

□ A □ B □ C □ D □ E □ F □ G □

Here □ represents the possible locations for those 5 girls.

Therefore number of ways of stand those 7 boys = $7!$

Arranging 5 girls in 8 locations $= {}^8P_5$

Total number of ways = $7!\times {}^8P_5$

(c) Since M (= Mathematics books)-5 , C (=Chemistry books)-4, P (=Physics books)-3. Total number of books=12.

(i) $12!$

(ii) Arranging 5 M 's $= 5!$, Arranging 4 C 's $= 4!$, Arranging 3 P 's $= 3!$

$\therefore$ Total number of ways $= 3!\ (5!\times 4!\times 3!)$.

$$A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \text{ and}$$

$$A^3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 5 & 5 \\ 5 & 2 & 5 & 2 \\ 5 & 5 & 4 & 5 \\ 5 & 2 & 5 & 2 \end{bmatrix}$$

2 paths which are: $v_2 \to v_3 \to v_1 \to v_4$ and $v_2 \to v_1 \to v_3 \to v_4$

(ii) $$A + A^2 + A^3 = \begin{bmatrix} 7 & 7 & 8 & 7 \\ 7 & 4 & 7 & 4 \\ 8 & 7 & 7 & 7 \\ 7 & 4 & 7 & 4 \end{bmatrix}$$

Here all the entries are non-zero. Therefore G is connected.

(iii) No!

G has cycle: $v_1 \to v_4 \to v_3 \to v_2 \to v_1$

(iv) H is not a component. H is connected but it is contained in another, larger, connected subgraph of G. It is shown below: