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**The Open University of Sri Lanka**  
**B.Sc. Degree Programme – Level 05**  
**Closed Book Test (CBT) - 2010/2011**  
**Pure Mathematics / Computer Science**  
**PMU 3294/ CSU 3276/ PME 5294 – Discrete Mathematics**



**Duration: - One & Half Hours**

**Date: - 21-10-2010.**

**Time: - 4.00 p.m. – 5.30 p.m.**

**Answer All Questions.**

1. (a) Let the number plate of a vehicle consists of four different letters followed by four digits. Find
    - (i) the total number of number plates that can be made,
    - (ii) the number of number plates in which the first digit cannot be 0,
    - (iii) the number of number plates in which no digit repeats.
  - (b) In how many ways can seven boys and five girls stand in a line so that no girls are next to each other.
  - (c) Find the number of ways in which five Mathematics books, four Chemistry books and three Physics books can be placed on a shelf if
    - (i) there are no any restrictions,
    - (ii) all books of the same subject are together.
  - (d) In how many ways can the fifteen students take three different tests if five students are to take each test.
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2. (a) Cricket teams Australia and England play for Ashes cup. But the probability of the team Australia wins a match is  $0.6$ . If the tournament is scheduled so that the team that first wins two of three matches wins the Ashes cup and the tournament is over.
    - (i) Draw a tree diagram to find the number of possible ways in which the tournament can occur,
    - (ii) Find the probability that the team England wins the Ashes cup.

(b) Let  $A$  and  $B$  be two events with  $P(B) > 0$ . Define  $P(A/B)$ , the conditional probability of  $A$  given  $B$ . Find  $P(A/B)$  if

- (i)  $B$  is a subset of  $A$ ,
- (ii)  $A$  and  $B$  are mutually exclusive,
- (iii)  $A$  and  $B$  are independent.

(c) In a survey of 100 students 55 like cricket, 38 like football and 23 like both cricket and football. What is the probability that a student randomly selected from this group likes exactly one of cricket or football.

3. (a) What is the largest possible number of vertices in a graph with 30 edges if all the vertices have degree at least 3.

(b) Let  $G$  be a graph with set of four vertices  $\{v_1, v_2, v_3, v_4\}$ , whose adjacency matrix  $A$  is given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

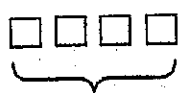
- (i) Determine the number of paths of length three joining  $v_2$  and  $v_4$ ,  
What are those paths?
- (ii) Deduce that  $G$  is connected,
- (iii) Is  $G$  a tree? Justify your answer,
- (iv) The subgraph  $H$  of  $G$  is defined by  
 $V(H) = \{v_1, v_3, v_4\}$  and  $E(H) = \{\{v_1 v_3\}, \{v_1 v_4\}, \{v_3 v_4\}\}$ ,  
Determine whether  $H$  is a component of  $G$ . Justify your answer.

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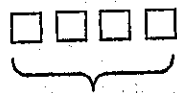


**Model Answer**

01. (a)



Different letters



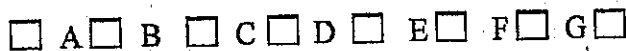
digits can repeat

(i)  $26 \times 25 \times 24 \times 23 \times 10 \times 10 \times 10 \times 10$

(ii)  $26 \times 25 \times 24 \times 23 \times 9 \times 10 \times 10 \times 10$

(iii)  $26 \times 25 \times 24 \times 23 \times 10 \times 9 \times 8 \times 7$

(b) Let A, B, C, D, E, F, G are 7 boys.



Here ☐ represents the possible locations for those 5 girls.

Therefore number of ways of stand those 7 boys =  $7!$

Arranging 5 girls in 8 locations =  ${}^8P_5$

Total number of ways =  $7! \times {}^8P_5$

(c) Since  $M$  (=Mathematics books)-5,  $C$  (=Chemistry books)-4,

$P$  (=Physics books)-3. Total number of books=12.

(i)  $12!$

(ii) Arranging 5  $M$ 's =  $5!$ , Arranging 4  $C$ 's =  $4!$ , Arranging 3  $P$ 's =  $3!$

$\therefore$  Total number of ways =  $3! (5! \times 4! \times 3!)$ .

$$A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \text{ and}$$

$$A^3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 5 & 5 \\ 5 & 2 & 5 & 2 \\ 5 & 5 & 4 & 5 \\ 5 & 2 & 5 & 2 \end{bmatrix}$$

2 paths which are:  $v_2 \rightarrow v_3 \rightarrow v_1 \rightarrow v_4$  and  $v_2 \rightarrow v_1 \rightarrow v_3 \rightarrow v_4$

$$(ii) \quad A + A^2 + A^3 = \begin{bmatrix} 7 & 7 & 8 & 7 \\ 7 & 4 & 7 & 4 \\ 8 & 7 & 7 & 7 \\ 7 & 4 & 7 & 4 \end{bmatrix}$$

Here all the entries are non-zero. Therefore  $G$  is connected.

(iii) No!

$G$  has cycle:  $v_1 \rightarrow v_4 \rightarrow v_3 \rightarrow v_2 \rightarrow v_1$

(iv)  $H$  is not a component.  $H$  is connected but it is contained in another, larger, connected subgraph of  $G$ . It is shown below:

