THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF INDUSTRIAL STUDIES

TTZ5244 – QUANTITATIVE TECHNIQUES

FINAL EXAMINATION 2011/2012

DURATION – THREE HOURS

Date: 22nd March 2012 **TIME: 1400 -1700 hrs.**

Answer 05 questions including Question 01 which is compulsory. All questions carry 20 marks amounting to total of 100.

You should clearly show the steps involved in solving problems.

No marks are awarded for the mere answers without writing the necessary steps

Question (01)

(i) Solve the following equations **(02 Marks)**

(a) (i) $2^{3x+1} \times 8^{x} = 64$ (b) $5^{3x+5} + 5 = 3130$

(ii) Express the following expressions with x as the subject. **(02 Marks)**

(a) $y = A(1 + r)^{2x}$ (b) $(4 + y)^{3x} = 12$

(iii) What do you understand by $\left[\frac{dy}{dx}\right]$, if y is a function of x? **(02 Marks)**

(iv) Differentiate the following functions with respect to x **(02 Marks)**

(a) $y = 10x^6 + 8x^5 + 2x^3 + 5x^2 - 24$ (b) $y = e^{4x+2}$

(v) Determine the stationary points of the function $y = 12x^3 - 4x + 5$ **(02 Marks)**

(vi) Determine the second derivative of the function $y = 4x^3 + 5x^2 + 6x$ **(02marks)**

(vi) Define **"inverse of a matrix"**. **(02 Marks)**

(vii) If there are two matrices A and B, such that

$$A = \begin{pmatrix} 3 & 2 & 5 \\ -1 & 3 & 4 \\ 2 & 1 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 & 1 \\ -3 & -7 & 8 \\ 5 & 2 & 0 \end{pmatrix}$$

Compute AB. **(03 marks)**

(viii) Find the determinant of the following matrix A. **(03 marks)**

$$A = \begin{pmatrix} 3 & 2 & 5 \\ -1 & 3 & 4 \\ 2 & 1 & -3 \end{pmatrix}$$

(02). (i). Illustrate how linear equations are used in modelling. **(04 Marks)**

(ii). Desktop publishing cost for a Statistics book amount to Rs. 2,000/=. In addition, Rs. 4/= per copy is incurred for printing the book. The publishers receive Rs 20/= a copy from the sale and pay 10% of this to the author. Write down an expression for publishers total profit in terms of the number of copies printed. Assume that all copies are sold. Sketch a graph of this expression. **(06 Marks)**

(iii). Describe how you would determine 'turning points' of a function. Also describe how to check whether they are a maxima or minima. **(04 Marks)**

(iv). Determine the position and nature of all turning points of the following function and hence sketch the graph. **(06 Marks)**

$$Y = \frac{X^3}{3} + \frac{X^2}{2} - 2X + 1$$

(03). (i). Differentiate the following functions with respect to X

(a). $Y = (3X - 5)(2X^2 + 6X + 10)$ (b). $Y = \log_e (5X^4 + 4X^2 + 8)$

(06 Marks)

(ii). The revenue R can be expressed as a function of Q (Quantity sold) as follows:

$R = Q(50 - 2Q) + 5$

(a). Find the derivative of R with respect to Q. **(02 Marks)**

(b). If the "marginal revenue function" is defined as $\frac{dR}{dQ}$, what is the marginal revenue when Q = 10? **(04 Marks)**

(iii). The price elasticity of demand (E) in economics is defined as follows:

$$E = - \frac{dQ}{dP} \frac{P}{Q}$$

where, P is the price and Q is the demand. What is the price elasticity of demand at P = 5, when the demand function is $Q = 200 - P^2 - 6P$ **(08 Marks)**

(04) (i). Determine the inverse of the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix}$ **(08 Marks)**

(ii). Hence solve following sets of equations.

$X + 2Y + Z = 4$

$3X - 4Y - 2Z = 2$

$5X + 3Y + 5Z = -1$ **(12 Marks)**

(05) (i). If $A = \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix}$

Determine following matrices, if they exist

(a). AB (b). BA **(05 Marks)**

(ii). Is it possible to determine the matrix AB for the following matrices. If not, why not?

If $A = \begin{pmatrix} 2 & 6 \\ 5 & 7 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$ **(02 Marks)**

(iii). If $A = \begin{pmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{pmatrix}$ determine A^T and $A.A^T$ **(05 Marks)**

(iv). If $A = \begin{pmatrix} -3, 2 \\ -2, 2 \end{pmatrix}$, $I = \begin{pmatrix} 1, 0 \\ 0, 1 \end{pmatrix}$ and $\underline{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Find the value of x and y so that $A^2 + xA - yI = O$ **(08 Marks)**

(06) The ABC Ltd., manufacturing refrigerators has at present, firm orders for the next 6 months. The company can schedule its production over 6 months to meet orders on either regular or overtime basis. The order size and production costs over the next six month are as follows.

Month	1	2	3	4	5	6
Orders	640	660	700	750	550	650
Cost / unit (Rs.) - Regular production	40	42	41	45	39	40
Cost / unit (Rs.) - Overtime production	52	50	53	50	45	43

With 100 refrigerators in stock at present, the company wishes to have at least 150 refrigerators in stock at the end of 6 months. The regular and overtime production in each month is not to exceed 600 and 400 units respectively. The inventory carrying cost for refrigerator is Rs 12/= per month. Formulate this problem as a liner programming model. **(20 marks)**

(07) A company produces two products A and B. The profit contribution of each product has been estimated as Rs 20/= for A and Rs 24/= for product B. Each product passes through three sections of the plant. The time required for each product and total time available in each department are as follows

Department	Product time for A per unit (Hrs)	Product time for B per unit (Hrs)	Available hours during the month
01	2	3	1,500
02	3	2	1,500
03	1	1	600

The company has a contract to supply at least 250 units of product B per month. Formulate the problem as a linear programming model.

i. What are the variables in this problem? **(02 marks)**

ii. What is the objective of this problem? **(02 marks)**

iii. What are the constraints of the problem? **(02 marks)**

iv. Solve the formatted programme **graphically** to determine the optimum product mix in order to maximise the profit. **(14 marks)**

(08) ABC Ltd. manufactures two products A and B. Three (03) units of raw material and 02 hours of labour are required to produce one unit of A, while one unit of raw material and one hour of labour is required to produce one unit of B. Contribution to profit are Rs 7/= per unit of A and Rs 5/= per unit of B. Forty eight (48) units of raw materials and 40 hours of labour are available. The company management team would like to determine the number of each product to be manufactured in order to get maximum profit.

(a) Formulate the problem as a Linear Programme model. **(06 Marks)**

(b) Determine using **simplex method,** how many of each products need to be manufactured in order to maximise the profit. **(14 Marks)**