00129

The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Final Examination - 2009/2010
Pure Mathematics / Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics-Paper I

Duration: - Two & Half Hours

Date: - 01-02-2010. **Time: - 9.30 a.m. – 12.00 noon.**

Answer FOUR questions only.

01. (a) Let p and q be two statements. Without using a truth table, show that $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to $\sim p$.

Verify the result by means of a truth table.

(b) Write down the negation of each of the following statements in clear and concise English. Do not use the expression "It is not the case that" in your answers.

(i) "For all integers a and b, there exist integers q and r such that $b = qa + r$",

(ii) "If a and b are odd natural numbers, then ab is an odd natural number and $a+b$ is an even natural number",

(iii) "The lengths of sides of a triangle are m^2+n^2, m^2-n^2, $2mn$ if and only if the triangle is right angled".

02. (a) Define *a tautology*.

(b) Let p, q and r be three statements. Use the method of conditional proof to show that $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$ is a tautology.

(c) Classify each of the following statements as *true* or *false* and explain your answers:

(i) "$4 \neq 2+2 \Rightarrow 7 < \sqrt{50}$",

(ii) "For every real number x, there exists an integer n such that $n \leq x < n+1$",

(iii) "For each real number x, there exists an integer y such that $xy = 1$".

03. Prove or disprove the following statements and name the method of your proof:

(i) "$n^2 + n$ is even for any integer n",

(ii) "If x is an irrational, then x^2 is irrational",

(iii) "$\forall n \in \mathbb{N},\ 1^3 + 2^3 + 3^3 + \ldots + n^3 = (1+2+3+\ldots+n)^2$",

(iv) "$\sqrt[3]{2}$ is a rational number".

04. (a) Let $A = \{1,2,3,4,5,6,7,8,9\}$. For $a, b \in A$, define ${}_aR_b$ iff ab is a perfect square, the square of an integer.

(i) Write down the ordered pairs in this relation,

(ii) For each $a \in A$, find $\overline{a} = \{x \in A \mid {}_xR_a\}$,

(iii) Determine whether the relation R is an equivalence relation.

(b) Let A be a set on which an equivalence relation R is defined. Then any two equivalence classes of A are either identical or disjoint.

(c) Consider the set $\mathbb{Z}$ of integers and integer $m > 1$. If $x - y$ is divisible by m, show that this defines an equivalence relation on $\mathbb{Z}$.

05. (a) Define the Cartesian product of two non-empty subsets A and B of a universal set U.

(b) Let A and B are two non-empty subsets of a universal set U, prove that $(A\times B)\cap(A\times C)=A\times(B\times C)$.

(c) Determine which of the following relations are total orders:

(i) Let R be a relation defined on A by ${}_aR_b$ if "x divides y" for each $a, b\in A$, where $A=\{1, 2, 3, 4, 6\}$,

(ii) Let R be a relation defined on $P(X)$, the power set of X, by if $A\subseteq B$ for each $A,B\in P(X)$,

(iii) Let R be a relation defined on $\mathbb{R}$ by ${}_xR_y$ if $x\leq y$ for each $x,y\in\mathbb{R}$.

Justify your answer.

06. (a) Determine whether the operation "*subtraction*" in $\mathbb{Z}$, the set of integers, is associative.

(b) Let G be a group and if $a,b\in G$, prove that the equation $ax=b$ has unique solution in G.

(c) Let G be the group of real numbers under addition, and let G' be the group of positive real numbers under multiplication. Show that the mapping $f:G\rightarrow G'$, defined by $f(a)=2^a$, is a homomorphism.

Is it an isomorphism? Justify your answer.