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The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Final Examination - 2009/2010
Pure Mathematics / Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics-Paper II

Duration: - Two & Half Hours

Date: - 01-02-2010. **Time: - 1.00 p.m. – 3.30 p.m.**

Answer FOUR questions only.

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01. (a) A computer password consists of a letter of the alphabet followed by three or four digits. Find

(i) the total number of passwords that can be created,

(ii) the number of passwords in which no digit repeats.

(b) How many bit strings of length ten either begin with three 0's or end with two 0's.

(c) In how many ways can ten adults and five children stand in a line so that no children are next to each other?

(d) Three consecutive coefficients of a Binomial expansion of $(1+x)^n$ are 45, 120 and 210. Find the value of n.

02. (a) A woman has 11 colleagues in her office, of whom 8 are men. She would like to have some of her colleagues to dinner. Find the number of choices if she decides to invite all her women colleagues and sufficient men colleagues to make the number of women and men equal.

(b) Teams A and B play in a cricket tournament. But the probability of the team A wins a game is 0.6. If the team that first wins two of three games wins the tournament.

(i) Draw a tree diagram to find the number of possible ways in which the tournament can occur.

(ii) Find the probability that the team B wins the tournament.

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(c) There are 20 mathematics major students and 15 computer science major students in the discrete mathematics class. Find the probability of selecting 12 students to form a committee from the class, if

(i) all must be mathematics major,

(ii) the two discipline must have the same number of representatives.

03. (a) Let A and B be two events with $P(A) > 0$. Define $P(B/A)$, the conditional probability of B given A.

(b) Find $P(B/A)$ if,

(i) A is a subset of B,

(ii) A and B are mutually exclusive.

(c) In a certain college 25% of the students failed mathematics, 15% of the students failed computer science, and 10% of the students failed mathematics and computer science. A student is selected at random.

(i) If he failed computer science, what is the probability that he failed Mathematics,

(ii) What is the probability that he failed mathematics or computer science,

(iii) Determine whether the event failed mathematics is depend on the event failed computer science.

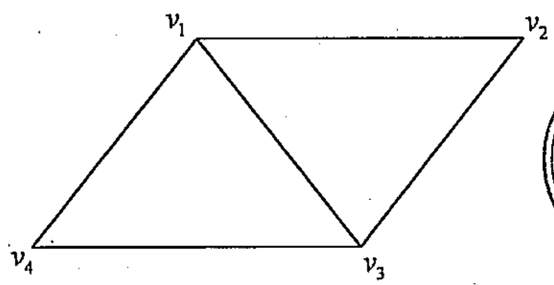
04. (a) Define the degree and parity of a vertex of a graph.

(b) Prove that $\sum_{i=1}^{n} \delta(v_i) = 2\times(\text{Number of edges in the graph})$.

$\left(\text{Hint: } \delta(v_i) = \sum_{j=1}^{n} a_{ij}\right.$ where n is the number of vertices of the graph

and a_{ij} is the $(i,\ j)^{th}$ entry of the adjacency matrix of the graph$\left.\right)$

(c) Give the set theoretic definition of the following graph G:



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(i) By using the above graph G, verify the result in part (b),

(ii) Write down the adjacency matrix of the graph G. Hence determine the number of paths of length three joining v_2 and v_4.

What are those paths?,

(iii) Deduce that G is connected,

(iv) Is G a tree? Justify your answer.

05. (a) Suppose the graph $G = G(V, E)$ has five vertices. Find the maximum number m of edges in E. Give an example of a graph such that every vertex is adjacent to two vertices and every edge is adjacent to two edges.

(b) Let G be the graph in the following figure:

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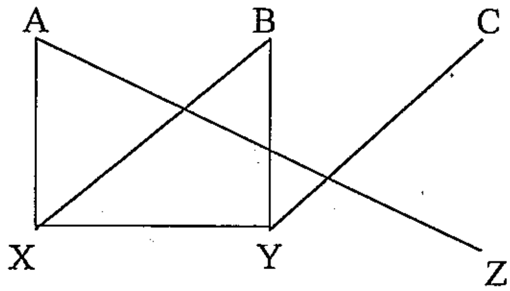

(i) Determine whether or not each of the following sequence of edges forms a path:

(a) $\{\{A,X\},\{X,B\},\{C,Y\},\{Y,X\}\}$ (b) $\{\{X,B\},\{B,Y\},\{Y,C\}\}$

(c) $\{\{A,X\},\{X,Y\},\{Y,Z\},\{Z,A\}\}$ (d) $\{\{B,Y\},\{X,Y\},\{A,X\}\}$

Justify your answer.

(ii) Find all single paths from A to C and all cycles in the graph G.

(iii) The subgraph H of G is defined by

$V(H)=\{A, C, Z\}$ and $E(H)=\{\{A, Z\}, \{A, X\}, \{X, Y\}, \{Y, C\}\}$.

Determine whether H is a component of G.

06. (a) A person invests Rs 2000 at 15 percent interest compounded annually. If A_n represents the amount at the end of n years, find

(i) a difference equation satisfied by A_n and initial conditions that define the sequence $\{A_n\}$,

(ii) an explicit formula for A_n. Hence, deduce that, how long will it takes for the person to double the initial investment?

(b) Find the general solution of the difference equation $f(n+2)-4f(n)=n(1+3^n)$.