Waves and Vibrations –PHU 2141-Level 4
The Open University of Sri Lanka- Department of Physics
B.Sc. Degree Programme -2010/2011

Assignment

The answer scripts showed be submitted on or before 15th October 2010

(1) (a)Using the series expansion of the sine and cosine factions, prove that

$e^{j(\omega t + \varphi)} = \cos(\omega t + \varphi) + j\sin(\omega t)$

(b) Express $Z = 4\sin(\omega t) + 3\cos(\omega t + \varphi)$ in the form of $Z = \mathrm{Re}(A e^{j(\omega t + \varphi)})$

(2) A mass connected to a spring is initially at its equilibrium position ($x = 0$) and then it oscillates simple harmonically with an amplitude of 5 cm at a frequency of 1 Hz.

(a) Find the possible equations describing the position of the mass as a function of time, in the form $x(t) = A\cos(\omega_o t + \varphi)$. What are the numerical values of A, ω_o, and φ?
(b) What are the values of x, dx/dt, and d^2x/dt^2 at $t = 3$ sec?

(3) The equation of an point is given as

$x(t) = 2\sin(\pi/2\, t + \pi/4)$. Find

(a) the period of the oscillations
(b) the maximum velocity of the point
(c) its maximum acceleration

(4) An object of mass 0.2 kg is hung from a spring whose spring constant is 80 N m^{-1}. The body is subject to a resistive force of magnitude bv, where v is its velocity (m s^{-1}) and $b = 4$ N m^{-1} s.

(a) Set up the differential equation of motion for free oscillations of the system, and find the period of such oscillations.

(b) The object is subjected to a sinusoidal force given by $F(t) = F_0 \cos \omega t$ where $F_0 = 2$ N and $\omega = 30$ rad s^{-1}. In the steady state, what is the amplitude of the forced oscillation?

Instead of a driving force (see (b)), we now oscillate the end of the spring at the top end vertically with a harmonic displacement $X(t) = X_0 \cos \omega t$.

(c) Set up the differential equation of motion for this driven oscillator.

(d) What is the amplitude of the mass in steady state for $\omega = 0$, 30 and 300 rad s^{-1} ? Say that $X_0 = 0.5$ cm in all cases.

(5) An electronic circuit in a musical instrument is capable of producing sinusoidal signals with different frequencies and amplitudes and also it can superimpose them to produce a single periodic wave pattern given by the following equation ;*V(t)*.

$V(t) = t \qquad (0 < x < \pi/2)$
$V(t) = \pi - t \qquad (\pi/2 < x < \pi)$

(a) State the mathematical theorem (Fourier series) that can be applied with this phenomena.
(b) Sketch the wave pattern given by the above equation.
(c) Write down the fundamental frequency of the sinusoidal signal used in the above circuit to produce the given signal.
(d) Find the amplitudes and frequencies of first five sin signals required to produce the above voltage signal.
(e) Using them show that how the superimposed signal approaches to given wave pattern.
(f) State the Fourier integral theorem in detail and compare it with the Fourier series theorem

(6) (a) Calculate the observed frequencies under the Doppler effect at the following situations

Velocity of the sound source (ms^{-1})	**Velocity of the listener (ms^{-1})**
0	+ 25
0	- 200
-75	+ 25
75	-300

(Note : " +" velocity indicates the direction from sound source to listener)

(b) A jet plane flies overhead at an altitude of 1400 m. The sound intensity on the ground as the jet passes overhead is 170 dB. At what altitude should the plane fly so that the ground noise is no greater than 177 dB, the threshold of pain? Ignore the finite time required for the sound to reach the ground.

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Prepared by Vijanaka Fernando/Dept: Physics-OUSL-2010-06-17