The Open University of Sri Lanka
B.Sc. Degree Programme Level-04
Final Examination – 2009/2010
PHU 2141/PHE 4141 – Waves and Vibrations/ Geometrical Optics
Duration : Two and a half (2 ½) hours

Date: 12th January 2010 **Time: 9.30 a.m.- 12.00 noon**

Answer Four (4) questions only

(1) Suppose that an object with mass m = 0.3 kg is free to move in one dimension with its position at any time given by the function $y(t)$. The mass is subjected to a force given by $F(t)=-ky(t)$, where k = 1.2 N m^{-1}.

(i) Starting with the principles, show that the position of the mass, undergoing a simple harmonic motion may be described by $y(t) = A\cos(\omega_o t + \varphi)$ where A , ω_o and φ are constant. Then determine the angular frequency (ω_o) of the motion substituting the given values.

(ii) Suppose, at t = 0, the mass is at y = 2 m moving with a speed of −1.5 m s^{-1}. Determine the amplitude of the motion and then figure out the first time after t = 0, the object is stationary. (You might, or might not, find this useful : $\tan^{-1}0.375 = 0.36$ rad)

(2) A progressive simple harmonic wave travelling in a string is given by $y(x,t)=10\cos(0.1x-4.0t)$ where y and x are expressed in centimetres and time in seconds.

Calculate the following parameters of the wave motion.

(i) Amplitude, angular frequency(ω), velocity , angular wave number(k) and wavelength with their units.

(ii) Maximum transverse speed and acceleration of a particle in the spring.

(iii) Derive the standing wave equation considering the reflected wave at the end of the string. . Sketch the wave.

(iv) Show that the group velocity ($d\omega/dk$) of the standing wave is zero.

(3) An object of mass m= 0.2 kg is hung from a spring whose spring constant is k=80 Nm^{-1}. The body is subjected to a resistive force of magnitude $-bv$, where b = 4 N m^{-1}s and v is its velocity (ms^{-1}).

(i) Set up the differential equation of motion ,$x(t)$ for free oscillations of the system.

(ii) Obtain the solutions for the motion $x(t)$ and find the period of such oscillations.

(Hint: the solution of differential equation; $\frac{d^2x(t)}{dt^2}+\gamma\frac{dx(t)}{dt}+\omega_0^2x(t)=0$

is in the form of $x(t) = Ae^{-\gamma t/2}\cos(\omega t+\alpha)$ where $\omega^2=\omega_o^2-\gamma^2/4$ and A and α are constants)

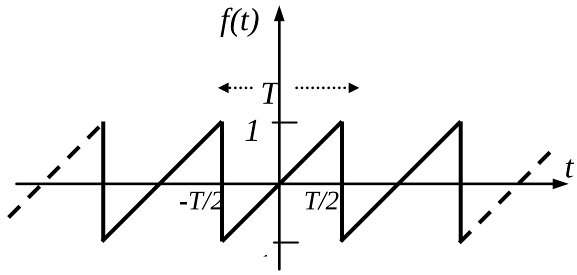
Sketch the wave.

(iii) If the mass is critically damped(ie $\omega_o=\gamma/2$)and subject to the initial conditions ;$x(x=0)=0$ and $v(t=0)=-4ms^{-1}$, find the maximum amplitude of the oscillations.

(iv) Find the total mechanical energy of the system at the given time t, under the condition at (iii).

(4) The following graph shows the one dimensional oscillations of an air particle with the time, $f(t)$.

(i)Find the equation for the function $f(t)$ with in the time period T.



(ii) Find the Fourier series for the function $f(t)$.

(You might, or might not, find these integrals useful: $\int x\sin(ax)dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a}$)

(iii) Write down the amplitudes of fundamental frequency and its first five harmonics. Hence sketch the frequency spectrum.

(5) The following two waves in a medium are superimposed: $y_1(x,t) = 0.02\sin(9.2x - 18t)$ and $y_2(x,t) = 0.02\sin(11.0x - 24t)$ where x is in meters and t is in seconds.

(i) What are the phase velocities of wave $y_1(x,t)$ and $y_2(x,t)$?
(ii) Derive the equation for the superimposed wave ;$y(x,t)=0.04\sin(10.1x - 21t)\cos(0.9 - 3)$
(you may use $p\sin(A) + p\sin(B) = 2p\sin\{(A+B)/2\}.\cos\{(A-B)/2)\}$
(iii) Sketch the oscillation pattern of particles in the medium.
(iv) Find the group velocity and phase velocity of the superimposed wave.
(v) What is the length of one "group" of waves?

(6) A train travelling along a straight track at 108 km h-1 emits a whistle of frequency 1000 Hz.

(i)What frequency is heard by
(a) the engine driver,
(b) an observer ahead of the train,
(c) the same observer after the train has passed?

(ii) A jet plane flies overhead at an altitude of 1200 m. The sound intensity on the ground as the jet passes overhead is 180 dB. At what altitude should the plane fly so that the ground noise is no greater than 150 dB, the threshold of pain? Ignore the finite time required for the sound to reach the ground.