The Open University of Sri Lanka
B.Sc. Degree Programme – Level 4
Advanced Electromagnetism - PHU 2142 / PHE 4142
Open Book Test – 2009/2010

MODEL ANSWERS

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1. (a) We know that, the energy, U, stored in a capacitor of capacitance C at a potential difference V between the plates is, $U = \frac{1}{2}CV^2$

The above equation can be expressed, with the term Q, as: $U = \frac{Q^2}{2C}$ *(because $Q = CV$)*

The above equation can be expressed, with the term A, as: $U = \frac{Q^2 d}{2\varepsilon_0 A}$ *(because $C = \varepsilon_0 A/d$)*

where d is the separation between the plates.

(b) Now the capacitor is isolated (i.e. no change in Q) and plates are pulled apart by a distance e.

Now the energy stored in the capacitor, with a plate separation $(d + e)$ is, $U^1 = \frac{Q^2(d+e)}{2\varepsilon_0 A}$

The extra energy is, $U^1 - U = \Delta U = \frac{Q^2 e}{2\varepsilon_0 A}$

This extra energy is due to the work done in pulling the plates apart against the attractive force between the plates and hence the work done is = $\frac{Q^2 e}{2\varepsilon_0 A}$

(c) The work done in pulling the plates apart through a distance, e, against the force of attraction, F, between the plates, can also be expressed as $F \times e$.

The work done is, $\Delta U = F \times e = \frac{Q^2 e}{2\varepsilon_0 A}$

Therefore, the force of attraction is = $F = \Delta U / e = \frac{Q^2}{2\varepsilon_0 A}$

(d) This is a case of a coaxial cable with the inner conductor of radius a and the outer screen conductor of radius b.

The energy density at a distance r is, $\frac{\varepsilon_o E(r)^2}{2} = \frac{\varepsilon_o}{2}\left(\frac{\lambda}{2\pi\varepsilon_o r}\right)^2$

$$U(r) = \frac{\lambda^2}{8\pi^2\varepsilon_o r^2}$$

This energy density symmetrically varies with the radius r from the axis of the cable.

Therefore, to find the total energy in this symmetrical situation, consider the energy stored in a small element of volume of $2\pi r \times dr \times dl$ and then take the volume integral, over the volume enclosed by the length l and the annular region from a to b.

Therefore, the total energy, U, is = $\int_0^l \int_a^b U(r) 2\pi r dr dl$

The total energy stored in the electric field is, $U = \int_0^l \int_a^b \frac{\lambda^2}{8\pi^2 \varepsilon_o r^2} 2\pi r dr dl$

$$= \frac{\lambda^2}{4\pi\varepsilon_o} \int_0^l dl \int_a^b \frac{1}{_o r} dr$$

$$= \frac{\lambda^2 l}{4\pi\varepsilon_o} \int_a^b \frac{1}{_o r} dr$$

$$= \frac{\lambda^2 l}{4\pi\varepsilon_o} \ln\left(\frac{b}{a}\right)$$

2. (a) The surface charge density σ of the given thin plastic disk is = $\frac{q}{\pi R^2}$

Then the charge between the radii r and $r + dr$ is,

$dq \quad = \sigma \, 2\pi r \, dr$

(b) Current (di) formed by this charge as the disk rotates is,

$di \quad = dq / T$ where T is period of rotations and $T = 2\pi / \omega$

(c) The magnetic field (dB) produced by this current (di) produces at the centre of the disk is,

$dB \quad = \mu_o \, di / 2r$

(d) The magnetic moment produced by the current di is, $d\eta = di \, \pi r^2$

Therefore, the total magnetic moment of the disk is,

$$\eta \quad = \int d\eta = \int_o^R \frac{\sigma 2\pi r}{2\pi / \omega} \pi r^2 \, dr$$

$$= \pi\sigma\omega \int_o^R r^3 \, dr = \frac{1}{4} \pi\sigma\omega . R^4$$

(e) (i) The magnetic field due to all charge is,

$$\text{B} \quad = \int_o^R dB = \int_o^R \frac{\mu_0 di}{2r} dr$$

$$= \int_o^R \frac{\mu_0}{2r}\frac{dq}{T}dr = \int_o^R \frac{\mu_0}{2r}\frac{\sigma 2\pi r}{2\pi/\omega}dr$$

$$= \frac{1}{2}\mu_0\sigma\omega\int_o^R dr = \frac{1}{2}\mu_0\sigma\omega R$$

$$= \frac{1}{2}\mu_0\omega R \times \frac{q}{\pi R^2}$$

$$= \frac{\mu_0\omega q}{2\pi R}$$

Students are requested to get the above formula starting from $\frac{\mu_0}{4\pi}\eta\frac{2}{z^3}$

3. (a) The mutual inductance of the ring-solenoid system is,

M = Magnetic flux through the ring / current in the solenoid

$$M = \frac{\mu_0 n_1 i \pi R_1^2}{i} = \mu_0 \pi n_1 R_1^2$$

(b) The induced emf in the ring is = $M\frac{di}{dt}$

But for steady current, di/dt = 0

Therefore, induced emf = 0.

(c) The magnetic flux through the area between the two wires is due to wire 1 is,

$$\Phi_{B1} = \int_0^l \int_a^{d-a} \frac{\mu_0 i}{2\pi r} dr dl$$

$$= \frac{\mu_0 i}{2\pi} l \ln\frac{d-a}{a}$$

Similarly, $\Phi_{B2} = \frac{\mu_0 i}{2\pi} l \ln\frac{d-a}{a}$

Inductance is,

$$L = \frac{\Phi_B}{i} = \frac{\Phi_{B1}+\Phi_{B2}}{i}$$

$$= \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$$

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