THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. DEGREE PROGRAMME - Level 4 - 2009/2010
DEPARTMENT OF PHYSICS
PHU 2142 / PHE 4142: Advanced Electromagnetism

Assignment – 1 **Due date: 26th March 2010**

Answer All Questions

1. Let ***A*** and ***B*** be vector functions of (x, y, z) and let Φ be a scalar function of (x, y, z).

(a) Prove that the divergence of a curl is always zero. {i.e. $\nabla.(\nabla \times A) = 0$}

(b) Prove that the curl of a gradient is always zero. {i.e. $\nabla \times (\nabla \Phi) = 0$}

(c) What does the expression ***(A.∇) B*** mean? (i.e. its x, y, and z components in terms of the Cartesian components).

(d) Compute $(\hat{r}.\nabla)\hat{r}$, where $\hat{r} = (x\hat{x} + y\hat{y} + z\hat{z})/\sqrt{x^2 + y^2 + z^2}$

2. For a point charge ($\underline{\mathbf{E}} = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r}$):

(a) Calculate $\nabla \times \underline{\mathbf{E}}$.

(b) Compute $\int_a^b E \bullet dl$, in spherical coordinates using $dl = dr\ \hat{r} + rd\theta\ \hat{\theta} + r \sin(\theta)\ d\Phi\ \hat{\phi}$

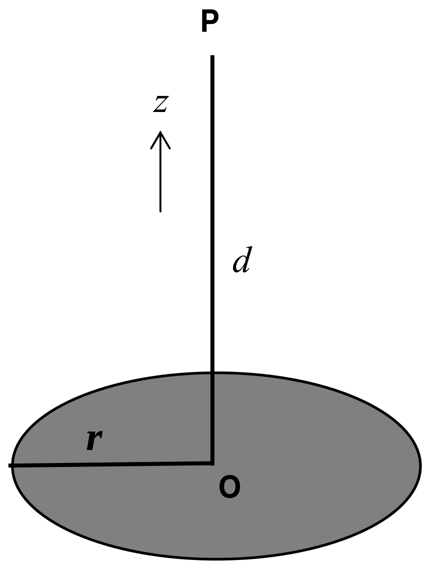
(c) Show that the integral around any closed path is zero.

(d) Compute $\nabla \times \underline{\mathbf{E}}$ from (c) by applying Stokes' theorem.

(e) Show how you can conclude from (a) or (d) that $\nabla \times \underline{\mathbf{E}} = 0$, for any static charge distribution.

3. A disk of radius r carries a uniform surface charge density σ. Thus the total charge, Q, on the disk is $Q = \pi r^2 \sigma$. The z – axis passes through the centre O, as shown in the diagram.

(a) What is the electric field $\underline{\mathbf{E}}$ (magnitude and direction) at the point P, at a distance d above the centre of the disk? Express your answer $\underline{\mathbf{E}}$ (z) in terms of Q, r, ε_o and d.



(b) Plot $\underline{\boldsymbol{E}}$ (z) as a function of z for all positive z's. Use r as your unit on the abscissa and use Q / $(4\pi\varepsilon_o r^2)$ as your unit for $\underline{\boldsymbol{E}}$ (z).

(c) Using Gauss's law, calculate $\underline{\boldsymbol{E}}$ (z) near point O, assuming $d << r$.

4. (a) State the Biot-Savart law which gives the magnetic field produced by a current element at a distance $\underline{\mathbf{r}}$ from the element.

(b) Using Biot-Savart law, find:

(i) the magnetic field at a distance 'r' from a long straight wire carrying a current 'I'.

(ii) the magnetic field at a distance 'b' along the axis of a circular current loop of radius 'a' and carrying a current I.

(c) A circular coil of radius 5 cm has 10 turns and carries a current of 5 ampere. Find the magnetic field at the center of the coil.

5. A long straight solid cylindrical conducting wire with radius R carries a steady current I.

(a) Calculate the magnetic field energy inside a length l of the wire.

(b) What is the contribution of the interior portion of the conductor to the total self-inductance?

(c) A coil with resistance 0.05 Ω and self-inductance 0.09 H is connected across a 12 V car battery of negligible internal resistance.

(i) How long after the switch is closed will the current reach 95 percent of its final value?

(ii) At that time how much energy (in Joules) is stored in the magnetic field?

(iii) How much energy has been delivered by the battery up to that time?

6. Maxwell's equations in free space (where $\underline{J} = 0,\ \rho = 0$) are:

$$\nabla \cdot \underline{E} = 0 \qquad \nabla \cdot \underline{B} = 0 \qquad \text{(Set 1)}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \nabla \times \underline{B} = \mu_o \varepsilon_o \frac{\partial \underline{E}}{\partial t} \qquad \text{(Set 2)}$$

(a) Using the above equations derive the wave equation for $\underline{\boldsymbol{B}}$.

(b) Show that the divergences of the second set equations are consistence. You may use the standard vector field identity $\nabla \bullet \nabla \times \underline{\mathbf{A}} = 0$, which holds for any vector field $\underline{\mathbf{A}}$.

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