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THE OPEN UNIVERSITY OF SRI LANKA

DEPARTMENT OF PHYSICS

BACHELOR OF SCIENCE DEGREE PROGRAMME -2011/2012 - LEVEL 05

FINAL EXAMINATION - 2011/12

PHU 3142 / PHE 5142 – PHYSICAL BASIS OF QUANTUM THEORY, THEORY OF RELATIVITY AND SOLID STATE PHYSICS

TIME: TWO HOURS (2 hrs) **ANSWER FOUR QUESTIONS ONLY**

Date : 19th December 2011 **Time : 1.00 pm to 3.00 pm**

You may assume that, mass of electron $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J s, $\pi = 3.14$, $\hbar = 1.05 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m s^{-1}, 1 eV = 1.6×10^{-19} J.

1) Briefly explain the Compton effect in Quantum Mechanics?

(a) Show that for Compton scattering, the angle ϕ at which the electron recoils is related to the angle θ of the scattered photon by

$$\tan\phi = \frac{\cot\left(\frac{\theta}{2}\right)}{1+\frac{E_0}{m_0c^2}}$$

Where E_0 is the energy of the initial photon, m_0 is the rest mass of the electron and c is the velocity of light.

(b) Hence show that the energy lost of the photon after the scattering is given by

$$E_0 - E = \frac{\left(\frac{2E_0}{m_0c^2}\right)\sin^2\left(\frac{\theta}{2}\right)}{1+\left(\frac{2E_0}{m_0c^2}\right)\sin^2\left(\frac{\theta}{2}\right)}$$

2) An electron with energy E is incident on a potential step with height $U_0 > E$. The situation may be described by the 1-D time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

with potential:
$$U(x) = \begin{cases} 0 & \text{for } x < 0 \\ U_0 & \text{for } x \geq 0 \end{cases}$$

(a) If the wave function in the region x < 0 is, $\psi_1(x) = Pe^{ikx} + Qe^{-ikx}$, using the Schrödinger equation, determine the value of k.

(b) If the wave function in the region $x \geq 0$ is $\psi_2(x) = Re^{-i\alpha x}$, using the Schrödinger equation, determine the value of α.

(c) State the boundary conditions which should be imposed on the wave function described in part (a) and part (b).

(d) Why are these boundary conditions necessary?

(e) Using the boundary conditions on the wave function that you have mentioned in part (b), show that

$$\frac{P}{Q} = \frac{k + i\alpha}{k - i\alpha}$$

3) (a) Write down the Lorentz transformation equations. Using these equations derive expressions for

(i) the length contraction

(ii) time dilation and

(iii)velocity transformation in x-direction

(b) An airplane whose rest length is 40.0 m is moving at uniform velocity with respect to Earth, at speed of 630 m s^{-1}.

(i) By what fraction of its rest length is it shortened to an observer on Earth?

(ii) How long would it take, according to Earth clocks, for the airplanes clocks to fall behind by 1.00 μs?

4) (i) Using the relativistic expression for mass show that the kinetic energy of a particle, T, is given by $T = mc^2 - m_0c^2$, where m and m_0 are the relativistic mass and rest mass of the particle respectively.

(ii) If P and E are the momentum and energy of the particle show that

$$E^2 - P^2c^2 = m_0^2c^4.$$

(iii) A particle of rest mass M decays when at rest into two particles of rest masses m_1 and m_2. If T_1 and T_2 are the kinetic energies of these two decayed particles respectively, show that

$$T_1 = \frac{c^2}{2M}\left(M^2 + m_1^2 - m_2^2 - 2Mm_1\right) \quad \text{and}$$

$$T_2 = \frac{c^2}{2M}\left(M^2 + m_2^2 - m_1^2 - 2Mm_2\right)$$

5) Define Bravais lattice, Primitive unit cell, Conventional unit cell, Lattice constant and Basis.

Determine the relationships between the lattice parameter a and the atomic radius *r* for monoatomic *simple cube*, *bcc* and *fcc* structure.

Draw (101) and (111) planes in a cubic unit cell. Determine the Miller indices of the directions which are common to both the planes.

The lattice constant of an fcc lattice is 6.38 Å. Determine the

(i) distance between a corner atom at the base and the atom at the centre of the top face.

(ii) largest distance between two atoms in the cubic cell.

6) Derive Bragg's law of X-ray diffraction in crystal. Discuss a method of investigating the atomic structure of a single crystal using X-ray diffraction techniques.

The first order Bragg reflection of a beam of X-rays incident on a NaCl crystal is observed at a glancing angle of $9^0 55'$.

(i) Find the wavelength of X-rays used.

(ii) Determine the angles at which second and third order Bragg reflections occur.

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