



The Open University of Sri Lanka
Department of Chemistry
PSE 3117 - Mathematics for Chemistry and Biology 2011/2012
Assignment Test 2
1.5 hours

Date:- 3rd October 2011

Time:- 4.00 p.m. - 5.30 p.m.

Reg. No.

- * This is a Structured paper with seven questions. Answers to be written ONLY in the space provided.
- * Total marks awarded = 240. Attempt all the questions, all those scoring about 200 or more would be deemed to have scored 100%. Pro-rata marks will be awarded to others.
- * The use of a non-programmable electronic calculator is permitted.
- * Write down clearly your name and address in the space provided in the answer script.
- * Switch off mobile phones. Keep them outside.

Ques.No.	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

- * මෙම ව්‍යුහගත ප්‍රශ්න පත්‍රයේ ප්‍රශ්න 7ක් ඇත. සපයා ඇති ඉඩ ප්‍රමාණයේ පමණක් පිළිතුරු ලිවිය යුතුය.
- * සම්පූර්ණ ලකුණු ගණන 240 කි. සියළුම ප්‍රශ්නවලට පිළිතුරු සැපයීමට උත්සාහ කරන්න. ලකුණු 200ක් පමණ ලබා ගන්නා සිසුන්ට 100% ලකුණු හිමිවනු ඇත. අනෙකුත් සිසුන් සඳහා සමානුපාතිකව ලකුණු ප්‍රදානය කරනු ලැබේ.
- * ප්‍රකුමණය කළ නොහැකි ගණක යන්ත්‍ර භාවිතා කළ හැක.
- * පිළිතුරු පත්‍රයේ අවසාන පිටුවෙහි ඔබගේ නම සහ ලිපිනය පැහැදිලිව ලියන්න.
- * ජංගම දුරකථන ක්‍රියා වීරහිත කරන්න. ඒවා ලඟ තබා නොගන්න.

இவ்வினாத்தாள் ஏழு கட்டமைப்பு வினாக்களைக் கொண்டுள்ளது. தரப்பட்ட இடைவெளியில் மாத்திரம் விடைகள் எழுதப்பட வேண்டும்.

மொத்தப் புள்ளிகள் = 240. சகல வினாக்களையும் முயற்சிக்க, அண்ணளவாக 200 அல்லது அதற்கு மேற்பட்ட புள்ளிகளைப் பெறுவதற்கு 100% வழங்கப்படும்.

நெறிப்படுத்தப்படாத கணினியின் உபயோகம் அனுமதிக்கப்பட்டுள்ளது.

உமது பெயர், பதிவுஎண், முகவரி என்பவற்றைத் தெளிவாக விடைத்தாளின் இறுதிப்பக்கத்தில் எழுதவும்.

செல்லிடைப் பேசிகளை நிறுத்தி அவற்றை வெளியே வைக்கவும்.

1. Integrate. අනුකලනය කරන්න. தொகையிடுக.

$$(i) \int \frac{x}{3+x} dx$$

$$(ii) \int \left(3x^2 + \frac{2}{x} - 2 \right) dx$$

$$(iii) \int \sqrt{2x+3} dx$$

(iv) $\int x \cos x \, dx$

(40marks)

2. Evaluate the following integrals. පහත දැක්වෙන අනුකල අගයන්න.
பின்வரும் தொகையீடுகளைத் தீர்க்க.

(i) $\int_0^{\frac{\pi}{2}} 2 \sin 2\alpha \, d\alpha$

(ii) $\int_1^2 (2x + x^2 - 1) dx$

$$(iii) \int_a^b \frac{-2x}{1+x^2} dx$$

(iv) The average value of a function $f(x)$ in the region $x = a$ to $x = b$ is given by

$$\frac{\int_a^b f(x) dx}{b-a}$$

Find the average value of $x^2 - 2x + 1$ from 0 to 1

$x = a$ සිට $x = b$ දක්වා $f(x)$ ශ්‍රිතයක සාමාන්‍ය අගය $\frac{\int_a^b f(x) dx}{b-a}$ මගින් දෙනු ලබයි. 0 සිට 1 දක්වා $x^2 - 2x + 1$ ශ්‍රිතයේ සාමාන්‍ය අගය සොයන්න.

$x = a$ யிலிருந்து $x = b$ பகுதியில் $f(x)$ எனும் சார்பின் சராசரிப் பெறுமானம் $\frac{\int_a^b f(x) dx}{b-a}$ ஆகும். 0 - 1 இற்கு $x^2 - 2x + 1$ இனது சராசரிப் பெறுமானத்தைக் காண்க.

(40marks)

3. Solve the following differential equations. පහත දක්වා ඇති අවකලන සමීකරණ විසඳන්න.

பின்வரும் வகையீட்டுச் சமன்பாடுகளைத் தீர்க்க.

(i) $(x^2 - 1)y^3 dx + x^2 dy = 0$

(ii) $ye^x dx + (2y + e^x) dy = 0$

(30 marks)

4. (a) Three fair coins are tossed, write down the sample space.

Note: Each coin has two possible outcomes H (heads) and T (Tails).

සමබර කාසි තුනක් උඩ දමන ලදී. නියැදි අවකාශය ලියා දක්වන්න. සෑම කාසියකටම H හෝ T පමණක් ලබාදිය හැකි බව සලකන්න.

මூன்று ඉරේමාතීරියානු නාணයங்கள் சுண்டப்படுகின்றன. මාතීරී වෙළඳිකளை ගැනුණු. ඉව්වොරු නාණයමුම තලෙ (H), ටු (T) ඉකීය ඉරණු ආතීයමාන ටෙරුටෙරුකලෙකු කොණ්ඩරුකීරුතු.

(b) Find the probability that two heads are obtained. හීස් දෙකක් ලැබීමේ සම්භාවිතාව සොයන්න. ඉරණු තලෙකු ටෙරුවතරුකානු නීකුමුතකවෙකු කාණුක.

(30 marks)

5. Time taken (in seconds) by a group of 30 students to answer a simple mathematics

question is recorded as follows. සීසුන් 30 ක් සරල ගණිත ප්‍රශ්ණයකට පිළිතුරු දීමට ගත කළ කාලය (තත්පරවලින්) පහත දැක්වේ.

පීන්වරුම අඬ්ඬවණෙයීල් ඉරු ගණීය කණීත වීනාවීරුකු වීදෙයගීඬ්ඬතරුකු 30 මාණවරුකලෙකු කොණ්ඬ කුඬ්ඬමොණුරු සෙලවමුඬ්ඬත රේරුම (සෙකුකනීල්) ටතීචු සෙය්ඬ්ඬඬ්ඬණුණු.

22,	23,	24,	23,	30,	31,	11,	26,	10,	9,	30,	13
26,	14,	27,	10,	33,	12,	16,	22,	21,	28,	25,	27,
11,	15,	20,	19,	18,	17,						

(i) Group the above data by completing the following table. පහත දැක්වෙන වගුව භාවිතා කර ඉහත දත්ත සමූහගත කරන්න. කීරේ තරඬඬ්ඬ අඬ්ඬවණෙයීල් ටුරණඬ්ඬඬ්ඬතවතණු මුලම මේලේ තරඬඬ්ඬ තරඬුකලෙ වකුඬ්ඬකලෙතු තොකුකුකු.

(40 marks)

- 6.(a) To pick a winning 4 digit lottery number, a lottery commissioner has four urns. Each urn contains ten balls. The balls in each urn are numbered zero through nine. One ball is picked out of each urn to make up one digit of the winning number. What is the probability that the winning number consists only of nines or zeros?

සංඛ්‍යා 4 ක් සහිත ලොතරියේ දිනුම් අංකයක් ලබාගැනීම සඳහා එක් එක් පෙට්ටියේ අංකනය කරන ලද බෝල 10ක් සහිත පෙට්ටි 4 ක් භාවිතා කර අවශ්‍ය සංඛ්‍යා ලබාගැනීම සිදුකරයි. සෑම පෙට්ටියකම ඇති බෝල 0 සිට 9 දක්වා අංකනය කර ඇත. එක් එක් පෙට්ටියෙන් බෝලය බැගින් ඉවතට ගෙන බෝලයේ සටහන් කර ඇති අංකය දිනුම් අංකයේ සංඛ්‍යාවක් සඳහා යොදාගනු ලැබේ. 9 සහ 0 යන අංක වලින් පමණක් සෑදුන දිනුම් අංකයක් ලැබීමේ සම්භාවිතාව සොයන්න.

நான்கு இலக்கமுடைய அதிஷ்ட இலாப எண்ணை வென்றெடுப்பதற்கு அதிஷ்ட இலாப ஆணையாளர் நான்கு கூடுகளை வைத்திருந்தார் ஒவ்வொரு கூடும் 10 பந்துகளைக் கொண்டிருந்தது. ஒவ்வொரு பந்துகள் 1 தொடக்கம் 9 வரை இலக்கமிடப்படுகின்றன. வெல்லப்படுகின்ற எண்ணானது ஒவ்வொரு கூட்டிலிருந்து எடுக்கப்படும் பந்துவினது இலக்கத்திலிருந்தும் வெல்லப்படுகின்ற எண்ணானது இலக்கம் ஒன்பது மாத்திரம் அல்லது பூச்சிய இலக்கத்தை மாத்திரம் கொண்டிருப்பதற்கான நிகழ்தகவு என்ன?

- (b) In how many ways can the letters of the word "PROBLEM" be rearranged to make 7 letter words such that none of the letters repeat? කිසිදු අකුරක් එක් වරකට වඩා නොයෙදෙන සේ "PROBLEM" යන වචනයේ අකුරු ප්‍රතිසංවිධානය කිරීමෙන් කොපමණ ආකාරයකට අකුරු 7 ක් සහිත වචන සාදා ගත හැකි දැයි සොයන්න. "PROBLEM" எனனும் சொல்லினது எழுத்துக்களை எந்வொரு எழுத்தும் மீண்டும் நிகழாமல் 7 எழுத்துச் சொற்களை எத்தனை வழிகளில் மீள் ஒழுங்காக்கிப் பெறமுடியும்

The Open University of Sri Lanka
Department of Chemistry
PSE 3117 - Mathematics for Chemistry and Biology 2011/2012
Assignment Test 2

Answer guide

$$1. (i) \int \frac{x}{3+x} dx = \int \frac{x+3-3}{3+x} dx = \int dx - \int \frac{3}{3+x} dx$$

$$= x - 3 \ln(3+x) + C$$

$$(ii) \int \left(3x^2 + \frac{2}{x} - 2 \right) dx = \int 3x^2 dx + 2 \int \frac{1}{x} dx - 2 \int dx$$

$$= \frac{3x^3}{3} + 2 \ln x - 2x + C_2$$

$$(iv) \int x \cos x dx$$

Let $x = v$ and $\cos x dx = du$
 $u = \sin x$ and $dv = dx$

Using $\int v du = vu - \int u dv$
 $= x \sin x -$
 $= \underline{\underline{x \sin x + \cos x + C_4}}$

$$2. (i) \int_0^{\frac{\pi}{2}} 2 \sin 2\alpha d\alpha = \left| 2 \times \frac{1}{2} (-\cos 2\alpha) \right|_0^{\frac{\pi}{2}}$$

$$= -\cos \pi - (-\cos 0) = 1 + 1 = 2$$

$$(ii) \int_1^2 (2x + x^2 - 1) dx = \int_1^2 2x dx + \int_1^2 x^2 dx - \int_1^2 dx$$

$$\left| x^2 \right|_1^2 + \left| \frac{x^3}{3} \right|_1^2 - \left| x \right|_1^2 = 4 - 1 + \frac{8}{3} - \frac{1}{3} - (2 - 1)$$

$$= 3 + \frac{7}{3} - 1 = \underline{\underline{\frac{13}{3}}}$$

$$(iii) \int \sqrt{2x+3} dx$$

Let $u = 2x + 3$

$$du = 2dx \therefore dx = \frac{du}{2}$$

$$\int \sqrt{2x+3} dx$$

$$= \int u^{\frac{1}{2}} \frac{du}{2} = \frac{1}{2} \times \frac{2}{\frac{3}{2}} u^{\frac{3}{2}} + C_3$$

$$= \underline{\underline{\frac{1}{3} (2x+3)^{\frac{3}{2}} + C_3}}$$

$$(iii) \int_a^b \frac{-2x}{1+x^2} dx = - \left| \ln(1+x^2) \right|_a^b$$

$$= -\ln(1+b^2) + \ln(1+a^2) = \ln \frac{(1+a^2)}{(1+b^2)}$$

$$(iv) \text{ average value} = \frac{\int_0^1 (x^2 - 2x + 1) dx}{1-0}$$

$$= \left| \frac{x^3}{3} - x^2 + x \right|_0^1 = \frac{1}{3} - 1 + 1 - (0) = \underline{\underline{\frac{1}{3}}}$$

3. (i) $(x^2 - 1)y^3 dx + x^2 dy = 0$

$$(x^2 - 1)dx = -\frac{x^2 dy}{y^3}$$

$$\frac{x^2 - 1}{x^2} dx = -\frac{dy}{y^3}$$

$$\int dx - \int \frac{1}{x^2} dx = -\int \frac{1}{y^3} dy$$

$$x + x^{-1} = \frac{y^{-2}}{2} + C$$

$$\underline{\underline{x + \frac{1}{x} - \frac{1}{2y^2} = C}}$$

(ii) $ye^x dx + (2y + e^x) dy = 0$

$$Mdx + Ndy = 0$$

$$M = ye^x \quad N = (2y + e^x)$$

$$\left(\frac{\partial M}{\partial y}\right)_x = e^x$$

$$\left(\frac{\partial N}{\partial x}\right)_y = e^x$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \left(\frac{\partial M}{\partial y}\right)_x$$

exact differential equation

$$\int Mdx = \int ye^x dx = ye^x$$

$$\int Ndy = \int (2y + e^x) dy = \int 2y dy + \int e^x dy$$

$$= \frac{2y^2}{2} + e^x y$$

$$= y^2 + ye^x$$

$$\underline{\underline{\text{Solution: } y^2 + ye^x = C}}$$

4.(a)

(H,H,H),(H,H,T),(H,T,H),(H,T,T),(T,H,H),(T,H,T),(T,T,H),(T,T,T)

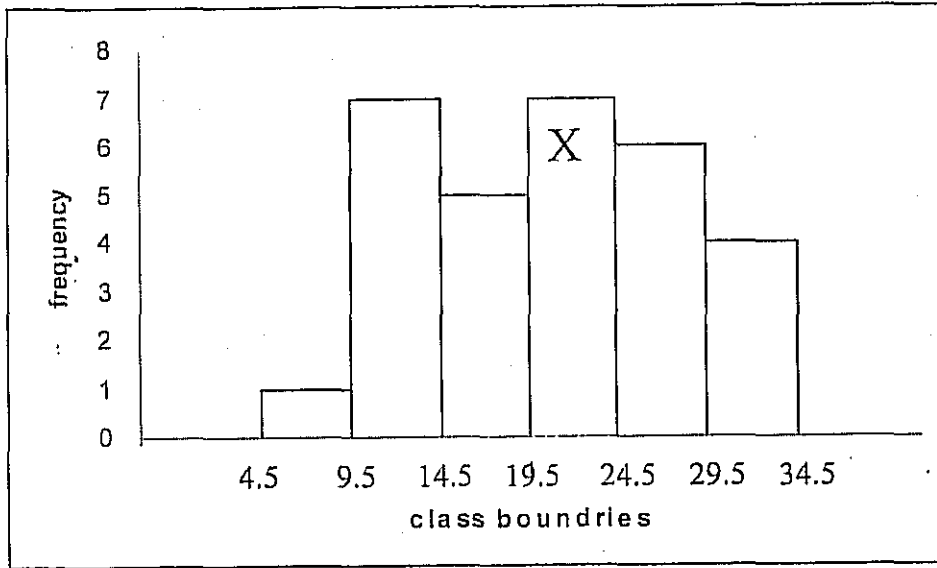
(b) Favourable outcomes (A) = (H,H,T),(H,T,H),(T,H,H)

$$P(A) = \frac{n(A)}{n(\epsilon)} = \frac{3}{8}$$

5. (i)

Class interval	Tally mark	Class mark x_i	Frequency f_i	$f_i x_i$	class boundaries
5 - 9		7	1	7	4.5-9.5
10 - 14	###	12	7	84	9.5-14.5
15 - 19	###	17	5	85	14.5-19.5
20 - 24	###	22	7	154	19.5-24.5
25 - 29	###	27	6	162	24.5-29.5
30 - 34		32	4	128	29.5-30.5

$$\sum f_i = 30 \quad \sum f_i x_i = 620$$



$$x = \frac{\sum f_i x_i}{\sum f_i} = \frac{620}{30} = 20.66$$

6.a (i) Prob. of drawing 9 as any digit = $\frac{1}{10}$

Since the winning number is a 4 digit number, the prob.

$$\text{that it consists of all 9s} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10000}$$

similarly, the prob. that the winning number consists of all '0's

$$= \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10000}$$

$$\text{The prob. that it consists of 9s or 0s} = \frac{1}{10000} + \frac{1}{10000} = \frac{1}{5000}$$

(i) දිනුම් අංකයේ යම් සංඛ්‍යාංකයක්

$$1 \text{ හෝ } 0 \text{ වීමේ සම්භාවිතාව} = \frac{2}{10}$$

මෙවැනි සංඛ්‍යාංක 4 ක් අන්තර්ගත නිසා
දිනුම් අංකය 1 සහ 0 සංඛ්‍යාංකවලින්

$$\text{සමන්විත වීමේ සම්භාවිතාව} = \frac{2}{10} \times \frac{2}{10} \times \frac{2}{10} \times \frac{2}{10} = \frac{16}{10000}$$

(b)

$$7! = \underline{\underline{5040}}$$

7. (a)

No. of ways to enter the room = 5

No. of ways to exit the room = 4

total number of ways = $5 \times 4 = \underline{20}$

(b) The probability of success = $\frac{2}{3}$

The probability of failure = $\frac{1}{3}$

$$\left(\frac{1}{3} + \frac{2}{3}\right)^5 = {}^5C_0 \left(\frac{1}{3}\right)^5 + {}^5C_1 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + {}^5C_2 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 + {}^5C_3 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 + {}^5C_4 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4 + {}^5C_5 \left(\frac{2}{3}\right)^5$$

$$\text{Probability that there will be four successes} = {}^5C_4 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^4 \cdot \frac{5}{3^5} = \underline{\underline{\frac{5}{243}}}$$