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THE OPEN UNIVERSITY OF SRI LANKA
B.SC. DEGREE PROGRAMME 2010/2011
FINAL EXAMINATION
PHU 3148/PHE 5148 - MATHEMATICAL PHYSICS
DURATION: TWO AND HALF (2 1/2) HOURS

Date: 30th June 2011 **Time: 1.30 pm -4.00 pm**

Answer **Four (4)** questions only

(1) For a scalar field $\phi\,(x,y,z)$ and a vector field $\underline{A}(x,y,z)$, obtain the following results.

(i) $\underline{\nabla}.(\phi\underline{A}) = \underline{\nabla}\phi.\underline{A} + \phi\underline{\nabla}.\underline{A}$

(ii) $\underline{\nabla}\wedge(\phi\underline{A}) = \underline{\nabla}\phi\wedge\underline{A} + \phi\underline{\nabla}\wedge\underline{A}$

Where, $\underline{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$

If $\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $|\underline{r}| = r$, n is a positive integer and $\underline{a}$, $\underline{b}$ are constant vectors. show that

(i) $\underline{\nabla}\, r^n = n\, r^{n-2}\, \underline{r}$

(ii) $\underline{\nabla}\,(\underline{r}.\underline{a}) = \underline{a}$

(iii) $\underline{\nabla}.(r^n\underline{r}) = (n+3)r^n$

(iv) $\underline{\nabla}.(\underline{\nabla}r^n) = n(n+1)r^{n-2}$

(v) $\underline{\nabla}\wedge(\underline{\nabla}r^n) = \underline{0}$

show also that,

(vi) $\underline{\nabla}\,[\underline{r}.(\underline{a}\wedge\underline{b})] = \underline{a}\wedge\underline{b}$

(vii) $\underline{\nabla}\wedge\left(\frac{\underline{a}\wedge r}{r^3}\right) = -\frac{\underline{a}}{r^3} + \frac{3\underline{r(a.r)}}{r^5}$

(2) (a) Solve the following linear equations by using the matrix method.

$$x + 2y = 4$$
$$3x - 5y = 1$$

(b) Find the Eigen values and Eigen vectors of the following matrix A.

$$A = \begin{pmatrix} 3 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

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(3) (a) State the necessary and sufficient conditions for the differential equation
M(x,y) dx + N(x,y) dy = 0 to be exact.
Determine a constant A such that the differential equation

$$\left(\frac{1}{x^2} + \frac{1}{y^2}\right) dx + \left(\frac{Ax+1}{y^3}\right) dy = 0 \text{ is exact.}$$

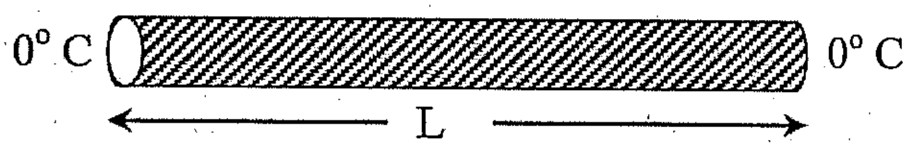
Solve the resulting exact equation.

(b) A chemical reaction converts a certain chemical into another chemical and the rate at which the first chemical is converted is proportional to the amount of the first chemical present at any time. At the end of one hour, 50 g of the first chemical remain, while at the end of three hours only 25 g of it remain.

(i) How many grams of the first chemical were present initially?

(ii) How many grams of the first chemical will remain at the end of the five hours?

(4) A cylindrical metal rod which is initially at the temperature of T_0 is completely covered with a insulator except both flat surfaces as shown in the following figure. Plunge the flat sides into ice water at temperature T = 0 °C.



The temperature; *T(x,t)* of the rod is modelled by the diffusion equation ;

$$\frac{\partial T(x.t)}{\partial t} = k\frac{\partial\ ^2T(x,t)}{\partial x^2} \quad (0 < x < L \text{ and } t > 0)$$

(a) Write down the initial conditions and the boundary conditions for T at the two ends of the rod at any time.

(b) Solve the differential equation and determine the expression for T(x,t).

(5) Define the Fourier series of a periodic function *f(x)* with period $2L$.

Find the Fourier series which represent the function *f(x) given* by

$$f(x) = \begin{cases} 0, & if -\pi \leq x < 0 \\ 1, & if \ 0 \leq x < \pi \end{cases}$$

Hence, show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(6) (a) Define the Laplace transformation.

(b) If $f(t)$ is continuous for $t > 0$ and exponential order as $t \to \infty$, while $f'(t)$ is sectionally continuous for $t > 0$ and if $L\{f(t)\} = F(s)$

(i) Show that $L\{f'(t)\} = sF(s) - f(0)$

(ii) Hence find $L\{f''(t)\}$

(iii) Using the above result, find $L\{t f''(t)\}$

(c) Solve the following differential-equation for $f(t)$ using the Laplace transformation.

$$f''(t) - 3f'(t) + 2f(t) = 2e^{-t}, \text{ if } f(0) = 2 \text{ and } f'(0) = -1$$

END