The Open University of Sri Lanka
B.Sc. Degree Programme-Level05
Department of Mathematics and Computer Science
CSU3275/PMU3293: Automata Theory
Final Examination -2010/2011
Duration: ~~Two & half~~ hours

Date: 17.08.2011 Time: 9.30am -12.30 pm.

Answer four questions only.

01. (a) Let x and y be any two strings over an alphabetΣ. Show that

$$|xy| = |x| + |y| \quad ; \text{where } |w| \text{ denotes the length of the string } w.$$

(b) Define a substring v of a string w.

Show that if u is a substring of a string w, then u^R is a substring of w^R, where x^R denotes the reversal of a string x. [Hint: you may assume that $(xy)^R = x^R y^R$ for any two strings x and y.

(c) State whether each of the followings are true or false. Justify your answer.

(i) $101 \in 0^*(0 \cup 1)^*$
(ii) $a^m b^m \in (ab)^*(ab)^m$; $m = 1, 2, ...$
(iii) $(a \cup b \cup ab \cup ba)^* = (a \cup b)^*$
(iv) $abb \in a^*c^*ba^*(bb)^*b^*a^*$

02. (a) Define a DFA (deterministic finite automaton) and describe the operation of it.

Design a DFA to accept the strings over {0, 1} consisting of an even number of 0s and an even number of 1s. Test your DFA with each of the following input strings. Clearly show the work you have done.

(i) 00
(ii) 10101
(iii) (00)*0110

(b) Define the language accepted by a DFA.

What is the language accepted by the DFA shown in Fig 2.1?

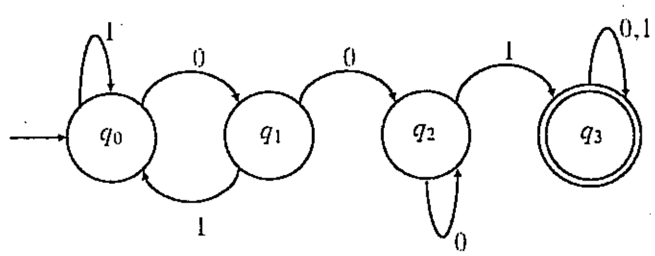

Fig 2.1

03. (a) Define an NFA (nondeterministic finite automaton) and describe the operation of it.

(b) What are the differences between a DFA and an NFA? Provide two applications of finite automata.

(c) Design an NFA to accept all strings over {0, 1} such that the third symbol from the right is 1.

(d) Providing an illustrative example, explain how you would transform a Moore machine to a Mealy machine.

04. (a) Explain what is meant by "implementation of a Mealy machine".

Let M = (S, I, O, δ, β) be a Mealy machine, where $S = \{s_1, s_2\}$, $I = \{i_1, i_2, i_3\}$, O = $\{o_1, o_2\}$, and whose state transitions and outputs are defined in Table 4.1.

	$\delta(s, i)$			$\beta(s, i)$		
	i_1	i_2	i_3	i_1	i_2	i_3
s_1	s_2	s_1	s_1	o_1	o_2	o_2
s_2	s_1	s_2	s_2	o_1	o_1	o_1

Table 4.1

In the usual notation, obtain M_{code} and M_{circ} machines.

(b) Define the isomorphism between two Mealy machines.

Let M_1, M_2 and M_3 be Mealy machines. Show that.

(i) $M_1 \approx M_1$
(ii) $M_1 \approx M_2$ and $M_2 \approx M_3 \Rightarrow M_1 \approx M_3$

05. (a) Define the behavioural equivalence between two Mealy machines.

Let M_1, M_2 and M_3 be Mealy machines. Show that

$$M_1 \equiv M_2 \text{ and } M_2 \equiv M_3 \Rightarrow M_1 \equiv M_3$$

(b) What is meant by a weaken homomorphism?

Let M_1 and M_2 be the Mealy machines defined in Table 5.1 and Table 5.2 respectively.

	$\delta(s, i)$		$\beta(s, i)$	
	a	b	a	b
p	q	r	1	2
q	p	r	1	2
r	q	s	2	1
s	p	s	2	1

Table 5.1 - M_1

	$\delta(s, i)$		$\beta(s, i)$	
	a	b	a	b
s_0	s_0	s_1	0	0
s_1	s_0	s_2	0	0
s_2	s_0	s_2	1	1

Table 5.2 – M_2

The triple $\phi = (\alpha, \sigma, \theta)$ is defined as $\alpha(p) = s_0$, $\alpha(q) = s_0$, $\alpha(r) = s_1$, $\alpha(s) = s_2$, and σ and θ are identity functions. Check whether ϕ: $M_1 \rightarrow M_2$ is a weaken homomorphism.

06. Explain what is meant by an SP partition of states of a Mealy machine.

Let M be the Mealy machine whose transitions and outputs are defined in Table 6.1.

	$\delta(s, i)$		$\beta(s, i)$	
	0	1	0	1
a	d	c	s	s
b	f	a	s	s
c	e	b	t	t
d	b	e	t	t
e	a	f	s	s
f	c	d	s	s

Table 6.1

Let $\pi = \{\{a, f\}, \{b, e\}, \{c, d\}\}$.

(i) Show that π is an SP partition of M.
(ii) Find another SP partition of M, different from π above, which consists of at least two elements and at most four elements.
(iii) Show that π is output consistent.
(iv) Construct the quotient machine $\frac{M}{\pi}$.