**The Open University of Sri Lanka**
**B.Sc/B.Ed Degree Programme**
**Final Examination- 2010/2011**
**Level 04- Applied Mathematics**
**AMU2185/AME4185 – Numerical Analysis I**

**Duration :- Two hours**

**Date:- 23.06.2011** **Time:- 9.30a.m.-11.30a.m.**

Answer **Four** questions only

**1. (a)** (i) Show by the graphical method that the equation $\cos x+1=x^2$ has two real roots in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(ii) By tabulating the values of $\cos x$ and $x^2-1$, obtain the numerical solutions of the above equation correct to 4 decimal places.

**(b)** A water tank in the shape of a right circular cylinder has been constructed. The radius and the height of the tank are $r=3.000m$ and $h=7.500m$. Find the volume $V$, correct to the appropriate decimal places.

**2. (a)** (i) Estimate the number of iterations that will be required to find a solution for $x+e^x=0$ in $[-1, 1]$ correct to 2 decimal places, by means of method of bisection.

(ii) Using the bisection method, find the root of $x+e^x=0$, in $[-1, 1]$ correct to 2 decimal places.

**(b)** Describe the geometrical interpretation of the Regula- Falsi formula for solving a non- linear equation $f(x)=0$.

**3. (a)** Let $x = g(x)$ be the re-arrangement of a given equation $f(x) = 0$. Then the iteration scheme with $x_0, x_1, \ldots, x_k$ in closed interval $R_1$ is given by $x_{k+1} = g(x_k)$. Show that the above scheme converges if $\max |g'(c)| < 1$, where $c \in (x_{k-1}, x_k)$.

**(b)** (i) Construct an iterative scheme that satisfies the condition for convergence to solve the equation $1 + \ln x - \frac{x}{2} = 0$.

(ii) Estimate the number of iterations that may be required for convergence to 3 decimal places.

**(c)** Using the above iterative scheme, find the root of the above equation correct to 3 decimal places.

**4. (a)** (i) Applying the Newton's method, find an iterative scheme that can be used to compute an approximate root of the equation $x^3 + 3 = 5x$.
Hence, find a root of the equation correct to 4 decimal places.

(ii) Discuss the advantages and disadvantages of using Newton's method.

**(b)** (i) Derive the Horner's scheme for division of a monic $n^{th}$ degree polynomial $f(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ by a factor $(x - \alpha)$.

(ii) Find all real roots of $x^3 - 5x + 3 = 0$. All roots should be correct up to 4 decimal places.

**5. (a)** Write the Lagrange interpolation polynomial $p(x)$ for the data set
$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$.

**(b)** Some values of the function $y = \cosh x$ are tabulated as follows:

| $x$ | 1.5 | 2 | 2.5 |
|---|---|---|---|
| $y$ | 2.352 | 3.762 | 6.132 |

(i) Find the Lagrange polynomial $y = p(x)$ through these points.

(ii) What is the maximum absolute error of using $p(x)$ in place of the given function.

(iii) Find the approximate value of $\cosh 2.3$.

**6. (a)** Derive the Newton's forward difference formula.

**(b)** The following table gives the population of a town during the last six censuses.

| Year | 1951 | 1961 | 1971 | 1981 | 1991 | 2001 |
|---|---|---|---|---|---|---|
| Population in thousands | 10 | 12 | 13 | 16 | 20 | 25 |

Estimate the population in 1966

**(c)** In the usual notation, prove the followings:

(i) $\nabla = 1 - E^{-1}$

(ii) $\frac{\nabla}{\Delta} = (1+\Delta)^{-1}$

(iii) $(1+\Delta) = (E-1)\nabla^{-1}$

(iv) $\left(E^{1/2} + E^{-1/2}\right)(1+\Delta)^{1/2} = 2+\Delta$