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The Open University of Sri Lanka
B.Sc. Degree Programme
Final Examination - 2010/2011
Applied Mathematics – Level 04
AMU2184/AME2184 –Newtonian Mechanics

Duration:- Two Hours

Date:- 27/06/2011 **Time:-1.30 p.m. - 3.30 p.m.**

Answer Four Questions Only.

01. A particle is projected vertically upwards from a fixed point O with speed u in a medium which offers a resistance $\frac{kv^2}{a+y}$ per unit mass, where v is the speed of the particle, y is the height above O and a and $k\left(\neq -\frac{1}{2}\right)$ are constants.

    Show that $(a+h)^{2k+1} = a^{2k+1}\left\{1+\frac{u^2(2k+1)}{2ag}\right\}$, where h is the maximum height attained by the particle.

    Considering the downward motion of the particle, write down equations to determine its velocity at time t.

02. A particle of mass m is at rest on a smooth horizontal plane and is connected by three elastic strings, each of modulus λ and natural length a to three points on the plane at the corners of an equilateral triangle of side $2\sqrt{3}\,a$. Prove that, if the particle is slightly displaced in the direction of one of the strings and then released, it will perform oscillations of period $\frac{4\pi}{3}\sqrt{(ma/\lambda)}$.

03. Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ with velocity $\underline{v}$ under force $\underline{F}(t)$, matter being emitted at a constant rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

A rocket of mass $M_0 + m_0$,where m_0 is the mass of fuel carried by the rocket, is launched vertically upwards. The fuel is ejected with a constant exhaust speed $\underline{u}$ and at a constant rate k with time. Neglecting the air resistance, write down the equation of motion of the rocket. Prove that when all the fuel is burnt out, the rocket will be at a height, $\frac{um_0}{k^2}\left[1+\ln\left(1+\frac{m_0}{M_0}\right)\right]-\frac{gm_0^2}{2k^2}$.

04. With usual notation show that the velocity and acceleration components of a particle moving in a plane, in plane polar coordinates, are given by $\underline{v}=\dot{r}\underline{e}_r+r\dot{\theta}\underline{e}_\theta$ and $\underline{a}=\left(\ddot{r}-r\dot{\theta}\right)^2\underline{e}_r+\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\underline{e}_\theta$.

A particle A of mass m, kept on a smooth horizontal table is connected by a light inextensible string passing through a small hole O in the table, to another particle B of mass $2m$ hanging from the other end. If A is projected horizontally with velocity $3\sqrt{ga}$ in a direction perpendicular to the string with OA=a, show that, in the subsequent motion, the length OA$=r$ satisfies the differential equation, $3\frac{d^2r}{dt^2}-\frac{9a^3g}{r^3}+2g=0$.

05. Show that, in the usual notation, the equation of the central orbit of a particle moving under a central force P per unit mass is given by $\frac{d^2u}{d^2\theta}+u=\frac{\text{P}}{h^2u^2}$ where $u=\frac{1}{r}$.

A particle P moving with a central force $\frac{\mu}{r^3}$ per unit mass directed towards the pole O is projected at a distance a from O with a velocity V perpendicular to the radius OP. Determine the orbits for the cases

(i) $\mu < a^2V^2$, (ii) $\mu = a^2V^2$ (iii) $\mu > a^2V^2$

06. (a) A particle starts from rest from an infinite distance from the earth and falls towards the earth only under its attraction. Show that the velocity of the particle on reaching a point at a distance R from the centre of the earth is given by $\sqrt{2GM/R}$, where M is the mass of the earth and G is the universal constant of gravitation.

(b) Derive an expression for the distance covered on earth by a geo-stationary satellite placed over the equator, and find this distance in kilometers.

(Radius of earth $R = 6336\,\text{km}$, $g = 9.81\,\text{ms}^{-1}$)