



*The Open University of Sri Lanka*  
Continuing Education / Stand Alone Courses in Science  
Final Examination 2011/2012  
*PSE 3117 - Mathematics for Chemistry and Biology*  
(2 hours)

20<sup>th</sup> December 2011

9.30 a.m. — 11.30 p.m.

- This paper has seven (7) questions. Total marks awarded = 220
- Attempt all the questions; all those scoring about 200 or more would be deemed to have scored 100%; pro rata marks would be awarded to other candidates.
- The use of a non-programmable electronic calculator is permitted.
- Clearly write down all relevant steps in answering the questions.
- $\log_e X = \ln X = 2.303 \log_{10} X$
- Show all relevant steps in arriving at your final answer.

- இவ்வினாத்தாள் ஏழு(07) வினாக்களைக் கொண்டுள்ளது. ஒதுக்கப்பட்ட மொத்த புள்ளிகள் = 220
- சகல வினாக்களையும் முயற்சிக்க. 200 அல்லது அதற்கு மேற்பட்ட புள்ளிகளைப் பெறுபவர்களுக்கு 100 % புள்ளிகள் வழங்கப்படும். ஏனையவர்களுக்கு விகிதாசாரப்படி புள்ளிகள் வழங்கப்படும்.
- நெறிப்படுத்தப்படாத கணனியின் உபயோகம் அனுமதிக்கப்பட்டுள்ளது.
- வினாக்களுக்கு விடையளிக்கையில் தேவையான சகல படிகளையும் தெளிவாக எழுதவும்.
- $\log_e X = \ln X = 2.303 \log_{10} X$

- 1 (a) Simplify and express the following in the form,  $a + ib$ .  
பின்வருவனவற்றை எளிதாக்கி அத்துடன்  $a + ib$  எனும் வடிவத்தில் தருக.

$$\frac{(2-3i)}{(2+i)} + (2i-1)^2$$

(12 புள்ளிகள்)

(b) Solve for  $x$ . /  $x$  இனைத் தீர்க்க:

(i)  $3^{2x} - 4(3^x) + 3 = 0$

(ii)  $3x + 4 + \frac{1}{x} = 0$  ( $x \neq 0$ )

(10 புள்ளிகள்)

2 (a) (i) Determine from first principles the values of  $A$  and  $B$  if

$$\log_{10} \frac{P \times Q^2}{R^3} = \log_{10} P + A - B$$

$\log_{10} \frac{P \times Q^2}{R^3} = \log_{10} P + A - B$  எனின் முதல் தத்துவங்களிலிருந்து  $A, B$  யினது பெறுமானங்களைத் தீர்மானிக்க.

(ii) Given that  $\log_{10} 5 = 0.6990$ , evaluate (without the aid of a calculator or log tables), the following.

$$\left[ \frac{[\log_2 10 + \log_2 25] \times \log_{10} 2}{\log_{10} \left( \frac{1}{100} \right)} \right] \left[ \frac{2}{3} \log_3 \sqrt{\frac{1}{27}} \right]$$

$\log_{10} 5 = 0.6990$  எனத் தரப்பட்டுள்ளது. பின்வருவதைத் (மடக்கை அட்டவணை அல்லது கணணியின் உதவியின்றி) தீர்க்க.

$$\left[ \frac{[\log_2 10 + \log_2 25] \times \log_{10} 2}{\log_{10} \left( \frac{1}{100} \right)} \right] \left[ \frac{2}{3} \log_3 \sqrt{\frac{1}{27}} \right]$$

(18 புள்ளிகள்)

(b) (i) Find the value of  $\theta$  ( $0 \leq \theta \leq 360$ ) that satisfies the equation.

$$2 \cos 2\theta + 4 \sin \theta - 3 = 0$$

பின்வரும் சமன்பாட்டை திருப்தியளிக்கும்  $\theta$  வினது பெறுமானத்தைக் ( $0 \leq \theta \leq 360$ ) காண்க.

$$2 \cos 2\theta + 4 \sin \theta - 3 = 0$$

(ii) Calculate a simplified value for C given that  $C = \frac{2 \cot 2\theta}{\cos \theta - \sin \theta} - \operatorname{cosec} \theta$

$$C = \frac{2 \cot 2\theta}{\cos \theta - \sin \theta} - \operatorname{cosec} \theta \text{ எனத் தரப்பட்டுள்ளது. C யிற்கான எளிய}$$

பெறுமானத்தைக் கணிக்க.

(12 புள்ளிகள்)

3. Find the first differential of the following functions: பின்வரும் சார்புகளின் முதலாம் வகையீடுகளைக் காண்க.

(i)  $v = \sqrt{x^2 - 2x + 1}$

(ii)  $y = \sec \theta \cdot \tan \theta$

(iii)  $y = 2x^2 \ln[2x + 1]$

(iv)  $v = \frac{2x - 3}{(x^2 - 1)}$

(32 புள்ளிகள்)

4. (a) Integrate the following/ பின்வருவனவற்றைத் தொகையிடுக.

(i)  $\int 2xe^x dx$

(ii)  $\int \sin 4\theta \cdot \cos 4\theta \cdot d\theta$

(iii)  $\int \frac{2x - 3}{(x - 1)(x - 2)} dx$

(iv)  $\int \frac{1 - q}{1 + q} dq$

(32 புள்ளிகள்)

(b) Evaluate/ தீர்க்க.  $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} \sin^2 \theta d\theta$

(08 புள்ளிகள்)

5. (a) The speed ( $V / \text{km h}^{-1}$ ) of an object is represented by the mathematical expression  $V = 8t^2 - 4t + 3$ , where t is time in hours.

பொருள் ஒன்றின் வேகம் ( $V / \text{km h}^{-1}$ ) பின்வரும் கணிதக் கோவையினால் பிரதிபலிக்கப்படுகின்றது.

$$V = 8t^2 - 4t + 3, \text{ இங்கு } t - \text{நேரம் (மணித்தியாலங்களில்)}$$

- (i) Calculate the time taken to record the lowest speed

மிகக் குறைந்த வேகத்தைப் பதிவு செய்வதற்காக எடுக்கப்படும் நேரத்தைக் கணிக்க.

- (ii) Calculate the distance traveled in the first 30 minutes (starting time,  $t = 0$ )

முதல் 30 நிமிடங்களில் (தொடக்க நேரம்  $t = 0$ ) பிரயாணித்த தூரத்தைக் கணிக்க.

(12 புள்ளிகள்)

- (b) Consider the function,  $U = f(x, y, z) = \frac{y^2 - z^2}{\sqrt{x}}$  எனும் சார்பைக் கருதுக

$$\text{Show that } \left[ \frac{\partial}{\partial z} \left( \frac{\partial U}{\partial x} \right) \right]_{z,y} \Big|_{x,y} = \left[ \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial z} \right) \right]_{x,y} \Big|_{z,y} \quad \text{எனக் காட்டுக.}$$

(12 புள்ளிகள்)

- (c) Show that  $f(P, T) = \frac{3P - 2T}{2P - 3T}$  is a homogeneous function and that, it satisfies the Eulers Theorem.

$$f(P, T) = \frac{3P - 2T}{2P - 3T} \quad \text{என்பது ஓர் ஓரிமமான சார்பு என்பதையும் அத்துடன் இது}$$

Eulers கொள்கையை திருப்திப்படுத்துகின்றது என்பதையும் காட்டுக.

(12 புள்ளிகள்)

6. (i) A card is drawn randomly from a deck of ordinary playing cards. You can win the game if the card is a spade or an ace. What is the probability that you will win the game?

சாதாரண விளையாடும் சீட்டுக் கட்டிலிருந்து எழுந்தமானமாக ஓர் சீட்டு இழுக்கப்படுகின்றது. கட்டிலிருந்து இழுக்கப்படும் சீட்டு ஓர் ஸ்பேட் (spade) அல்லது A (ace) ஆகக் காணப்படுமாயின் நீர் விளையாட்டில் வெற்றி பெறமுடியும். நீர் வெற்றி பெறும் நிகழ்தகவு யாது?

- (ii) In a certain country, the car number plate is formed by any 3 letters from the English alphabet followed by any 4 digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. How many different number plates can be formed?

சில நாடுகளில் காரினது இலக்கத்தகட்டின் எண்ணானது ஆங்கில அகர வரிசையிலிருந்து ஏதாவது மூன்று எழுத்துக்களினாலும் அவற்றைத் தொடர்ந்து 1, 2, 3, 4, 5, 6, 7, 8, 9 எனும் இலக்கங்களிலிருந்து ஏதாவது நான்கு இலக்கங்களினாலும் உருவாக்கப்படுகின்றது. எத்தனை வேறுபட்ட இலக்கத்தகட்டுகள் உருவாக்கப்படலாம்?

- (iii) A die is rolled and a coin is tossed; find the probability that the die shows an odd number and the coin shows a head.

தாயக்கட்டை ஒன்று உருட்டப்படுகின்றது அத்துடன் நாணயம் ஒன்றும் சுண்டப்படுகின்றது. தாயக்கட்டை ஒன்றை எண் ஒன்றையும், நாணயம் தலை ஒன்றையும் காட்டுவதற்கான நிகழ்தகவைக் காண்க.

- (iv) A coin is tossed 6 times. As the coin is an unbalanced one, probability of landing Head is  $\frac{2}{3}$ . What is the probability that it lands on Heads exactly three times?

நாணயம் ஒன்று ஆறு தடவைகள் சுண்டப்படுகின்றது. நாணயமானது சமனற்றதனால் தலை வீழ்வதற்கான நிகழ்தகவு  $\frac{2}{3}$  ஆகும். சரியாக மூன்று தடவைகள் தலை வீழ்வதற்கான நிகழ்தகவு யாது?

(32 புள்ளிகள்)

7. A class of 31 students provided the following data on the number of people living in their homes; the data is given below.

31 மாணர்களைக் கொண்ட வகுப்பொன்றின், மாணவர்களின் வீட்டில் வசிக்கும் ஆட்களின் எண்ணிக்கை கீழே தரவுகளாகத் தரப்பட்டுள்ளது.

வீட்டில் வசிக்கும் ஆட்களின் எண்ணிக்கை	2	3	4	5	6	7	8	9
மீட்டர்கள்	3	2	8	7	5	3	2	1

With the aid of a suitable table, / பொருத்தமான அட்டவணையொன்றின் உதவியுடன்

- (i) Calculate the mean number of people living in a house./ வீடு ஒன்றில் வசிக்கும் ஆட்களின் இடையைக் கணிக்க.
- (ii) Determine the median and mode of the above data./ மேற்கூறப்பட்ட தரவின் இடையம், ஆகாரம் என்பவற்றைத் தீர்மானிக்க.
- (iii) Calculate the variance of the data./ தரவினது மாறத்திறனைக் கணிக்க.
- (iv) Later one new student joined the class and his data was also added to the count. But the mean of the data did not change. How many people live in his house?  
அவ்வகுப்பில் மாணவனொருவன் தாமதமாக சேர்கின்றான். அவனது தரவும் கணக்கில் சேர்க்கப்பட்டது. ஆனால் தரவினது இடை மாற்றமடைய வில்லை. அம் மாணவனின் வீட்டில் எத்தனை ஆட்கள் வசிக்கின்றனர்?
- (v) Find the variance of the revised data.  
திருத்தப்பட்ட தரவினது மாற்றத்திறனைக் காண்க.

(24 புள்ளிகள்)

(பதிப்புரிமை பெற்றது)



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- Clearly write down all relevant steps in answering the questions.
- $\log_e X = \ln X = 2.303 \log_{10} X$
- Show all relevant steps in arriving at your final answer.

- මෙම ප්‍රශ්න පත්‍රයේ ප්‍රශ්න 7 ක් ඇත. සම්පූර්ණ ලකුණු ගණන 220 කි.
- සියලුම ප්‍රශ්නවලට පිළිතුරු සැපයීමට උත්සාහ කරන්න ලකුණු 200 ක් පමණ ලබාගන්නා සිසුන්ට ලකුණු 100 හිමිවනු ඇත. අනෙකුත් සිසුන් සඳහා සමානුපාතිකව ලකුණු ප්‍රදානය කරනු ලැබේ.
- ප්‍රකුමණය කළ නොහැකි ගණක යන්ත්‍ර භාවිතා කළ හැක.
- පිළිතුරු සැපයීමේදී අදාළ සියලුම පියවර පැහැදිලිව ලියන්න.
- $\log_e X = \ln X = 2.303 \log_{10} X$
- අවසාන පිළිතුර ලබාගැනීමට අදාළවන සියළුම පියවර දක්වන්න.

1 (a) Simplify and express the following in the form,  $a + ib$ . පහත දැ සුළුකර  $a + ib$  ආකාරයෙන් ප්‍රකාශ කරන්න.

$$\frac{(2-3i)}{(2+i)} + (2i-1)^2$$

(12 marks)

(b) Solve for x. x සඳහා විසඳන්න.

(i)  $3^{2x} - 4(3^x) + 3 = 0$

(ii)  $3x + 4 + \frac{1}{x} = 0 \quad (x \neq 0)$

(10 marks)

- 2 (a) (i) Determine from first principles the values of A and B if  $\log_{10} \frac{P \times Q^2}{R^3} = \log_{10} P + A - B$

$$\log_{10} \frac{P \times Q^2}{R^3} = \log_{10} P + A - B \text{ නම් ප්‍රථම මූල ධර්මවලින් A සහ B නිර්ණය කරන්න.}$$

- (ii) Given that  $\log_{10} 5 = 0.6990$ , evaluate (without the aid of a calculator or log tables), the following.

$\log_{10} 5 = 0.6990$  බව දී ඇත්නම් පහත දැක්වෙන දැඩි අගය සොයන්න.

$$\left[ \frac{[\log_2 10 + \log_2 25] \times \log_{10} 2}{\log_{10} \left( \frac{1}{100} \right)} \right] \left[ \frac{2}{3} \log_3 \sqrt{\frac{1}{27}} \right]$$

(18 marks)

- (b) (i) Find the value of  $\theta$  ( $0 \leq \theta \leq 360$ ) that satisfies the equation.

පහත දැක්වෙන සමීකරණය තෘප්ත කරන  $\theta$  ( $0 \leq \theta \leq 360$ ) අගයන් සොයන්න.

$$2 \cos 2\theta + 4 \sin \theta - 3 = 0$$

- (ii) Calculate a simplified value for C given that  $C = \frac{2 \cot 2\theta}{\cos \theta - \sin \theta} - \operatorname{cosec} \theta$

$$C = \frac{2 \cot 2\theta}{\cos \theta - \sin \theta} - \operatorname{cosec} \theta \text{ බව දී ඇත් නම් C සඳහා අගයක් සුළුකර දක්වන්න.}$$

(12 marks)

- 3 Find the first differential of the following functions: පහත දැක්වෙන ශ්‍රිතයන්ගේ ප්‍රථම අවකලය සොයන්න.

(i)  $v = \sqrt{x^2 - 2x + 1}$

(ii)  $y = \sec \theta \cdot \tan \theta$

(iii)  $y = 2x^2 \ln[2x+1]$

(iv)  $v = \frac{2x-3}{(x^2-1)}$

(32 marks)

4. (a) Integrate the following: පහත දැක්වෙන දෑ අනුකලනය කරන්න.

(i)  $\int 2xe^x dx$

(ii)  $\int \sin 4\theta \cdot \cos 4\theta \cdot d\theta$

(iii)  $\int \frac{2x-3}{(x-1)(x-2)} dx$

(iv)  $\int \frac{1-q}{1+q} \cdot dq$

(32 marks)

- (b) Evaluate. අගයන්න.  $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} \sin^2 \theta d\theta$

(08 marks)

5. (a) The speed ( $V/ \text{km h}^{-1}$ ) of an object is represented by the mathematical expression

$$V = 8t^2 - 4t + 3, \text{ where } t \text{ is time in hours.}$$

වස්තුවක වේගය ( $V/ \text{km h}^{-1}$ ),  $V = 8t^2 - 4t + 3$  යන ගණිතමය ප්‍රකාශනයෙන් දක්වා ඇත. මෙහි  $t$  යනු කාලය පැය වලිනි.

(i) Calculate the time taken to record the lowest speed. අවම වේගය සටහන් වීමට ගතවන කාලය ගණනය කරන්න.

(ii) Calculate the distance traveled in the first 30 minutes (starting time,  $t = 0$ ) පළමු මිනිත්තු 30 තුළදී ගමන් කළ දුර ගණනය කරන්න. (ආරම්භය  $t = 0$ )

(12 marks)

(b) Consider the function,  $U = f(x, y, z) = \frac{y^2 - z^2}{\sqrt{x}}$  යන ශ්‍රිතය සලකා බලන්න.

Show that  $\left[ \frac{\partial}{\partial z} \left( \frac{\partial U}{\partial x} \right) \right]_{x,y} = \left[ \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial z} \right) \right]_{x,y}$  බව පෙන්වන්න.

(12 marks)

(c) Show that  $f(P, T) = \frac{3P - 2T}{2P - 3T}$  is a homogeneous function and that, it satisfies the Eulers

Theorem.  $f(P, T) = \frac{3P - 2T}{2P - 3T}$  සමජාතීය ශ්‍රිතයක් බව සහ එය ඔයිලර් ප්‍රමේය තෘප්ත කරන බව පෙන්වන්න.

(12 marks)

(6) (i) A card is drawn randomly from a deck of ordinary playing cards. You can win the game if the card is a spade or an ace. What is the probability that you will win the game?

සාමාන්‍ය කාඩ් කුට්ටමකින් කොලයක් සමභාවිලෙස ලබා ගන්නා ලදී. එය ඉස්කෝප්ප කොලයක් හෝ ආසියා කොලයක් වුවහොත් ඔබට දිනුම හිමිවේ. ඔබ මෙම ක්‍රීඩාවෙන් ජය ලැබීමේ සම්භාවිතාව කුමක්ද?

(ii) In a certain country, the car number plate is formed by any 3 letters from the English alphabet followed by any 4 digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. How many different number plates can be formed? එක්තරා රටක කාර්වල අංක තහඩු සාදා ඇත්තේ ඉංග්‍රීසි හෝඩියේ ඕනෑම අක්ෂර තුනක් සහ ඊට පසු 1, 2, 3, 4, 5, 6, 7, 8 සහ 9 යන අංක අතුරින් ඕනෑම අතුරු 4ක් සම්බන්ධ කිරීමෙන්ය. මේ අන්දමට එකිනෙකට වෙනස් අංක තහඩු කොපමණ සෑදිය හැකිද?

(iii) A die is rolled and a coin is tossed; find the probability that the die shows an odd number and the coin shows a head. දාදු කැටයක් පෙරළන අතරතුර කාසියක්ද උඩ දමන ලදී. දාදු කැටයේ ඔත්තේ සංඛ්‍යාවක් සහ කාසියේ හිසක් ලැබීමේ සම්භාවිතාව සොයන්න.

(iv) A coin is tossed 6 times. As the coin is an unbalanced one, probability of landing Head is  $2/3$ . What is the probability that it lands on Heads exactly three times? කාසියක් 6 වරක් උඩ දමන ලදී. කාසිය සමබර නොවන නිසා හිසක් ලැබීමේ සම්භාවිතාවය  $2/3$  ක් වේ. තුන්වරක් පමණක් හිස ලැබීමේ සම්භාවිතාව කොපමණද?

(32 marks)



- (7) A class of 31 students provided the following data on the number of people living in their homes; the data is given below. සිසුන් 31 දෙනෙකු තම තමන්ගේ නිවාසවල වාසය කරන පුද්ගලයන් ගනන පිළිබඳ සැපයූ දත්ත පහත දක්වා ඇත.

No.of people living in a house	2	3	4	5	6	7	8	9
Frequency	3	2	8	7	5	3	2	1

With the aid of a suitable table, සුදුසු වගුවක් ආධාරයෙන්

- Calculate the mean number of people living in a house.  
නිවසක වාසය කරන පුද්ගලයන් සංඛ්‍යාවේ මධ්‍යයන අගය ගණනය කරන්න.
- Determine the median and mode of the above data.  
ඉහත දැක්වෙන දත්තයන්ගේ මධ්‍යස්ථය සහ මාතය සොයන්න.
- Calculate the variance of the data. දත්තවල විචලතාවය ගණනය කරන්න.
- Later one new student joined the class and his data was also added to the count. But the mean of the data did not change. How many people live in his house?  
පසුව පංතියට නවක ශිෂ්‍යයෙකු එකතු වූ අතර ඔහු විසින් සැපයූ දත්තද මෙයට එක් කරන ලදී. එහෙත් එවිටද දත්තයන්ගේ මධ්‍යයනයෙහි වෙනසක් සිදු නොවීය. ඔහුගේ නිවසේ වාසය කරන පුද්ගලයන්ගේ ගණන කීයද?
- Find the variance of the revised data. සංශෝධනය කළ දත්තවල විචලතාවය සොයන්න.

(28 marks)