

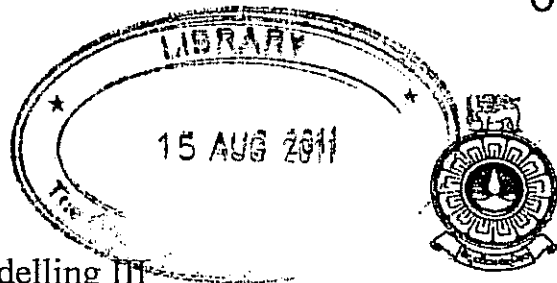
The Open University of Sri Lanka  
B.Sc. Degree Programme – Level- 04

Final Examination – 2010/2011

AMU 2183/AME 4183-Mathematical Modelling III

Applied Mathematics Level -04

Duration: - Two Hours.



Date: - 14-01-2011

Time: -9.30 a.m. – 11.30 a.m.

**Answer FOUR Questions only.**

01. Suppose that on January 1<sup>st</sup> a pair of baby rabbits is left on an island. These rabbits take two months to mature and on March 1<sup>st</sup>, they produce another pair of rabbits. They continually produce a new pair of rabbits on the first of every succeeding (following) month. Each new born pair takes two months to mature and produce a new pair on the first day of the third month of its life, and on the first day of every succeeding month. Show that the number of pair of rabbits on the island after  $n$  month is  $f_n$  assuming no

rabbits die, where  $f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n \sqrt{5}}$ ,  $n = 0, 1, 2, \dots$ ;  $f_0 = 0$ ;  $f_1 = 1$ ;  $f_2 = 1$

02. A customer requires a rectangular tank of volume  $V$  without the lid. The following are the costs put forward by a constructor;

- (i) cost per unit area of metal sheet is  $x$  rupees.
- (ii) cost per unit length of welding is  $2x$  rupees.

Find the dimensions of the tank that will minimize the construction cost and obtain the minimum cost.

Let  $V = 16$  units and  $x = 250$  rupees.

If the required tank can be purchased from a hardware shop at a cost of 15,000 rupees, determine whether the customer should buy the tank or construct it?

03. Imagine that there are 22 hostellers in your hostel including you and a cook. The hostellers themselves have weekly food committees, where the food committee is responsible for the supply of all food items during a period of one week. The purchases are determined by the food committee and made with funds provided by the 21 hostellers. While the cook gets free food and lodging. Suppose it is your turn to buy canned fish, where a can of variety A costs Rs. 21/= and a can of variety B costs Rs. 31/=. Your food committee gives you Rs 651/= with instruction that you should spend as much as possible and buy canned fish of both varieties. Show that you cannot spend all the above money but that it is possible to make a purchase with a saving of one rupee.

04. What are the assumptions made in the derivation of the formula, given in the usual notation,  $k \frac{d^2 y}{dx^2} = -M$  for bending of beams?

A uniform beam of weight  $W_1$  forms a horizontal bridge supported at its ends and a man of weight  $W_2$  stands on it at a point distant  $a, b$  from the ends. Show that the deflection under man's feet is,

$$y = \left( \frac{W_1 (a^2 + 3ab + b^2) + 8W_2 ab}{24k(a+b)} \right) ab,$$

where  $k$  is the flexural rigidity of the beam.

05. According to Newton's law of cooling, the rate of change of temperature of a body is proportional to the difference between the temperature of the body and temperature of the surrounding medium.

A mother boils a cup of milk to  $100^\circ C$  for her young child. The temperature of the milk drops to  $90^\circ C$  in 2 minutes in a room where the temperature is constant at  $20^\circ C$ . The child will drink only if its temperature is between  $35^\circ C$  and  $50^\circ C$ . If Newton's law of cooling applies, what is the maximum time period that the child can drink the milk?

06. Suppose you have Rs. 1000 to deposit in one of the five banks say, Bank M, Bank N, Bank O, Bank P or Bank Q. Suppose the money is to be left in the bank for 5 years and you want to maximize your ending balance. Bank M pays 15% simple interest per year; Bank N pays 13.5% interest compounded weekly. Bank O pays 14% interest compounded monthly; Bank P pays 13.3% interest compounded quarterly; and Bank Q pays 13% interest compounded continuously.

Which bank pays the most and which bank pays the least interest?

What is the effective interest rate of each of the banks?