THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. DEGREE PROGRAMME
LEVEL 04-APPLIED MATHEMATICS
OPEN BOOK TEST-2010/2011
APU 2144–Applied Linear Algebra and Differential Equations

DURATION: ONE AND HALF (1 ½) HOURS

Date: 23 March, 2011.   Time: 4.00 pm –5.30 pm

ANSWER ALL QUESTIONS.

1. (i) Suppose A is a matrix. When is A said to be invertible?

$$\text{Let } A = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}.$$

Find $|A|$.

Show that A is orthogonal.
Hence find
   (a) adj A.
   (b) A^{-1}.

2. (i) Explain the conditions for the consistency and the inconsistency of a system of equations.

(ii) Consider the following system of three linear equations.

$$x + 2y - 3z = -1$$
$$3x - y + 2z = 8$$
$$5x + 3y - 4z = 6$$

   (a) Let the coefficient matrix be denoted by A. Is A invertible?
   (b) Apply elementary row operations to solve the given system of linear equations.
   (c) Is the given system of linear equations consistent? Justify your answer.

3. (i) If A is the matrix

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

find an orthogonal matrix P such that $P'AP$ is a diagonal matrix where P' is the transpose of P.

(ii) By using Cayley Hamilton theorem compute the inverse of the non-singular matrix

$$\begin{pmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{pmatrix}$$