The Open University of Sri Lanka

B.Sc Degree Programme

Open Book Test (OBT)- 2010/2011

Level 04- Applied Mathematics

APU 2143 – Vector Calculus

Duration :- One and half hours

Date:- 08.09.2010 **Time:- 4.00p.m.-5.30p.m.**

Answer All Questions.

1. (a) State the domain and the range of the following functions.

(i) $f(x,y)=\sqrt{1-x^2-y^2}$

(ii) $g(x,y)=\sqrt{\dfrac{y-2}{x}}$

(b) Sketch the level curves of the above functions.

(c) Find the value of the following limits, if they exist.

(i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3-y^3}{x^3+y^3}$ (ii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+4}-2}$

2. (a) Discuss the continuity of the following function at (0, 0)

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$$

(b) Discuss the differentiability of the following function at (0, 0)

$$f(x,y)=\begin{cases}xy\dfrac{x^2-y^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$$

3. (a) Let $f(x, y, z) = x^3y + y^3z + z^3x$. Find a function $g(x, y, z)$ such that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = x^3 + y^3 + z^3 + 3g(x, y, z).$$

(b) Find the equation of the tangent plane to the surface $z = x^2 + 3y^2$ at the point $(1, 1, 4)$.

(c) Considering a suitable multivariable function estimate the value of $(1.98)^3\sqrt{(3.01)^2 + (3.97)^2}$.

(d) Find the critical points of the function $f(x, y) = 3x^2 + 12x + 8y^3 - 12y^2 + 7$ and determine their nature.