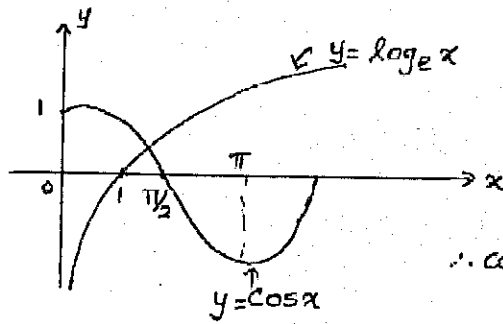


1. (a) There are some problems that can't be solved by using known analytical methods or otherwise.

(b) $\cos x = \log_e x$



$\therefore \cos x = \log_e x$ has a unique solution in the range $[1, \pi/2]$.

(c)

x	$\log_e x$	$\cos x$
1.00	0.5403	0
1.50	0.707	0.4055
1.40	0.1700	0.3365
1.30	0.2675	0.2624
1.29	0.2771	0.2546
1.31	0.2579	0.2700
1.302	0.2656	0.2639
1.303	0.2646	0.2647

\therefore root is 1.303.

2. (a)(i) Fundamental error - due to assumptions/ in the data

(ii) Rounding error - due to rounding a decimal number to a given number of decimal places.

(iii) Absolute error - $|e| = |x - \bar{x}|$, x - exact value, \bar{x} - computed value.

(b) $z = \frac{x}{y} = \frac{4.536}{1.32} = 3.4364$.

$|\Delta x| \leq \frac{1}{2} \times 10^{-3}$ & $|\Delta y| \leq \frac{1}{2} \times 10^{-3}$.

$z = xy^{-1}$

$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y} \Rightarrow |\Delta z| \leq \left[\left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y} \right| \right] \times z = \left(\frac{0.0005}{4.536} + \frac{0.0005}{1.32} \right) 3.4364 = 0.0133 \leq \frac{1}{2} \times 10^{-1}$

$\therefore z = 3.4$

The relative error of $z = \frac{|\Delta z|_{\max}}{|z|} = \frac{0.0133}{3.436} = 0.0039$

(c) $S = a + b$

$$r_a = \frac{e_a}{|a|}$$

$$r_b = \frac{e_b}{|b|}$$

$$r_s = \frac{e_s}{|s|}$$

e_a, e_b, e_s are errors in a, b & s .

r_a, r_b, r_s are relative errors in a, b & s .

$$e_s = e_a + e_b.$$

$$r_s = \frac{e_s}{|s|} = \frac{e_a + e_b}{|s|} = \frac{e_a}{|s|} + \frac{e_b}{|s|}$$

$$|r_s| \leq \left| \frac{e_a}{|s|} \right| + \left| \frac{e_b}{|s|} \right| = \left| \frac{r_a |a|}{|s|} \right| + \left| \frac{r_b |b|}{|s|} \right|$$

$$= \left| \frac{a}{s} \right| |r_a| + \left| \frac{b}{s} \right| |r_b|$$

3. (a) $f(x) = x^3 - x - 4 = 0$

$f(1) = -4, f(2) = 2$

$\therefore f(1) \cdot f(2) < 0$ \rightarrow root is lies between 1 & 2.

	x_0	x_1	x_2	$f(x_0)f(x_2)$	$ x_1 - x_0 $
0.	1	2	1.5	> 0	1
1.	1.5	2	1.75	> 0	0.5
2.	1.75	2	1.875	< 0	0.25
3.	1.75	1.875	1.8125	< 0	0.125
4.	1.75	1.8125	1.78125	> 0	0.0625
5.	1.78125	1.8125	1.796875	< 0	0.03125
6.	1.78125	1.796875	1.78906	> 0	0.0156
7.	1.78906	1.796875	1.79297	> 0	0.007815
8.	1.79297	1.796875	1.79492	> 0	0.003905
9.	1.79492	1.796875	1.79590	> 0	0.00195
10.	1.79590	1.796875	1.79639	< 0	0.000975
11.	1.79590	1.79639	1.796145		0.00049

\therefore root = 1.796

(b) $f(x) = 2x - \log_{10} x - 7 = 0$

$$x_{n+1} = \frac{x_{n-1} f_n - x_n f_{n-1}}{f_n - f_{n-1}}$$

n	x_{n-1}	x_n	f_{n-1}	f_n	x_{n+1}
1	3.5	4.0	-0.544068	0.397940	3.78878
2	4.0	3.78878	0.397940	-0.394 $\times 10^{-4}$	3.78928
3	3.78878	3.78928	-0.394 $\times 10^{-4}$	3.30 $\times 10^{-6}$	3.78928

\therefore root is 3.78928.