The Open University of Sri Lanka
B.Sc. Degree Programme –Level05
Department of Mathematics and Computer Science
Final Examination-2011/2012
CSU3275/PMU3293-Automata Theory
Duration: Three hours

Date: 01.12.2012	**Time: 1.30pm-4.30pm**

Answer FOUR questions only.

01. (a) Let Σ be an alphabet and w be any string over Σ. Explain the meaning of w^n for any positive integer n.

Show that $(w^n)^R = (w^R)^n$; where x^R denotes the reversal of string x. You may assume that $(xy)^R = y^R x^R$ for any two strings x and y. [**Hint:** use the mathematical induction on n.]

(b) Define a language L over an alphabet Σ.

Let L_1 and L_2 be the languages over $\Sigma = \{a, b\}$ defined by

$L_1 = \{ w \in \Sigma^* \mid w$ begins with a b and rest of the symbols (if exist) are a's$\}$
$L_2 = \{ w \in \Sigma^* \mid w$ consists of an odd number of b's$\}$

What is $L_1 \circ L_2$, the concatenation of L_1 and L_2? Justify your answer.

(c) Check whether the languages generated by each of the following pairs of expressions are identical or not. Justify your answer.

(i) $a(a^* \cup b^*)$ and $a(a \cup b)^*$
(ii) $(a(a^* \cup b^*))^*$ and $a(a \cup b)^*$
(iii) $a(a^* \cup b^*)^*$ and $a(a \cup b)^*$

02. (a) Define a deterministic finite automaton (DFA) and describe two applications of it.

Let M be a DFA and w be a string. Describe the operation of M when it is switched on with the string w on the input tape.

Prove or disprove $\delta^*(s, xy) = \delta^*[\delta^*(s, x), y]$; where x, y are any two strings and s is a state of M.

(b) Design a DFA to accept the language defined by

$L_1 = \{w \in \{0, 1\}^* \mid 110$ is a substring of $w\}$

Test your DFA with the following input strings.
(i) 011100
(ii) 1110^n1 ; n is a positive integer

00046

(iii) 110*

03. (a) What is a nondeterministic finite automaton (NFA)? Describe how it differs from a deterministic finite automation.

Define the language accepted by an NFA. Express, in natural language, the language accepted by the NFA given in Fig. 3.1?

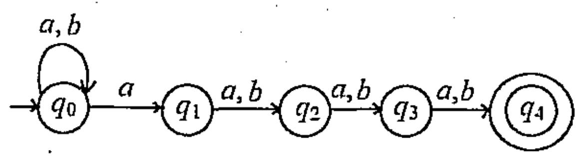

Fig 3.1

(b) Let M_1 be the Mealy machine defined in Table 3.1.

	$\delta(s, i)$		$\beta(s, i)$	
	0	1	0	1
a	a	b	s	t
b	b	a	t	t

Table 3.1

(i) Obtain a Mealy machine M_2 in such a way that M_1 is isomorphic to M_2.
(ii) Assume that M_1 and M_2 have been started in their corresponding states. Compare the behaviour of these two machines if the same input sequence is given to them.

04. (a) Define the behavioural equivalence between two Mealy machines.

Let M_1 and M_2 be two Mealy machines. Show that

(i) M_1 is behaviourally equivalent to itself.
(ii) If M_1 is behaviourally equivalent to M_2, then M_2 is behaviourally equivalent to M_1.

(b) Define the homomorphism of a Mealy machine into another Mealy machine.

Let M_1 and M_2 be two Mealy machines defined in Table 4.1 and Table 4.2 respectively.

	$\delta(s, i)$		$\beta(s, i)$	
	i_1	i_2	i_1	i_2
s_1	s_1	s_2	o_2	o_1
s_2	s_1	s_2	o_1	o_2

Table 4.1 – M_1

	$\delta(s, i)$		$\beta(s, i)$	
	j_1	j_2	j_1	j_2
t_1	t_2	t_1	p_2	p_1
t_2	t_2	t_1	p_1	p_2

Table 4.2 – M_2

Let the triple $\phi = (\alpha, \sigma, \theta)$ be defined by

$\alpha(s_1) = t_2,\ \alpha(s_2) = t_1$
$\sigma(i_1) = j_1,\ \sigma(i_2) = j_2$
$\theta(p_1) = o_2,\ \theta(p_2) = o_1$

Is ϕ a state behaviour assignment? Justify your answer.

05. Let $M_1 = (S_1, I_1, O_1, \delta_1, \beta_1)$ and $M_2 = (S_2, I_2, O_2, \delta_2, \beta_2)$ be two Mealy machines, and let κ be a function from O_1 to I_2. Define.

(i) The parallel composite $M_1 \| M_2$ of M_1 and M_2.
(ii) The serial composite $M_1 \oplus_\kappa M_2$ of M_1 and M_2 with respect to κ.

Let M be the Mealy machine defined in Table 5.1.

	$\delta(s, i)$		$\beta(s, i)$	
	0	1	0	1
a	a	b	p	q
b	b	a	p	p

Table 5.1

Construct $(M \| M) \oplus_\kappa M$, where $\kappa: \{p, q\} \times \{p, q\} \rightarrow \{0, 1\}$ defined by

$$\kappa(p, p) = 0 = \kappa(p, q),\ \ \kappa(q, p) = 1 = \kappa(q, q)$$

06. Define the SP partition of states of a Mealy machine.

Let M be the Mealy machine whose transitions and outputs are defined in Table 6.1.

	$\delta(s, i)$			$\beta(s, i)$		
	i_1	i_2	i_3	i_1	i_2	i_3
s_1	s_2	s_4	s_2	o_2	o_3	o_2
s_2	s_6	s_1	s_5	o_1	o_2	o_3
s_3	s_6	s_2	s_4	o_2	o_1	o_2
s_4	s_1	s_5	s_3	o_2	o_1	o_2
s_5	s_1	s_6	s_2	o_1	o_2	o_3
s_6	s_5	s_4	s_5	o_2	o_3	o_2

Table 6.1

Let $\pi = \{\{s_1, s_6\}, \{s_2, s_5\}, \{s_3, s_4\}\}$.

(i) Show that π is an SP partition of M.
(ii) Show that π is output consistent.
(iii) Find another SP partition of M, different from π above, which consists of at least three elements and at most four elements.
(iv) Construct the quotient machine $\frac{M}{\pi}$.