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The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX 6241 – Field Theory
Final Examination – 2013/2014

Date: 2014-08-13 Time: 0930-1230

Answer **five questions** by selecting **two from Section A, two from Section B** and **one from Section C**.

Section A

Select two questions from this section. (15 Marks for each)

Q1.

(a) Calculate the work done by moving a charge in an electric field of $\boldsymbol{E} = 2xy\,\boldsymbol{a}_x + (x^2 - z^2)\boldsymbol{a}_y - 3xz^2\boldsymbol{a}_z$ from $A(0,0,0)$to $B(2,1,3)$.

(b) Verify the identity:

$$\int \nabla \times \boldsymbol{A} \cdot \boldsymbol{dv} = -\oint \boldsymbol{A} \times \boldsymbol{dS}$$

for $\boldsymbol{A} = 5x^2y\,\boldsymbol{a}_x + 3xy^2\boldsymbol{a}_y$ and the volume defined by $0 < x < 2$, $-1 < y < 1$ and $-5 < z < 5$.

Q2.

(a) Explain the term "directional derivative".

(b) Given the scalar field $W = x^2y^2 + xyz$. Compute the directional derivative in the direction of $3\boldsymbol{a}_x + 4\boldsymbol{a}_y + 12\boldsymbol{a}_z$ at $(2,-1,0)$.

(c) Let U and V be scalar fields, show that

$$\oint_L U\nabla V \cdot \boldsymbol{dl} = -\oint_L V\nabla U \cdot \boldsymbol{dl}$$

Q3.

(a) Verify the following vector identities in Cartesian coordinates,

i) $\nabla \times (\nabla \times A) = \nabla(\nabla.A) - \nabla^2 A$

ii) $\nabla.\phi A = A.\nabla\phi + \phi\nabla.A$

(b) Green's theorem for two continuous functions f and g is written as

$$\iiint (f\nabla^2 g - g\nabla^2 f).dR = \iint (f\nabla g - g\nabla f).d\boldsymbol{S}$$

Evaluate

$$\int_C e^{-x}(\sin y\,dx + \cos y\,dy)$$

where C is the rectangle with vertices $(0,0)$, $(\pi,0)$, $(\pi,\frac{\pi}{2})$ and $(0,\frac{\pi}{2})$.

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Section B

Select **two** questions from this section. (20 Marks for each)

Q4.

(a) State the Gauss's law.

(b) A spherically symmetric charge distribution is given by

$$\rho_v = \begin{cases} \rho_0(1 - {}^{r}/_{a})^2 & ; r \leq a \\ 0 & ; r > a \end{cases}$$

Find $\boldsymbol{E}$ and V everywhere.

(c) Plot $\boldsymbol{E}$ and V along $\boldsymbol{a}_r$ and comment on it.

Q5.

(a) State under which conditions the Poison's equation becomes Laplace's equation.

(b) Conducting spherical shell with radii $a = 10\ cm$ and $b = 30\ cm$ are maintained at a potential difference of $100V$ such that $V(r = b) = 0$ and $V(r = a) = 100V$. Determine V and $\boldsymbol{E}$ in the region between the shells.

(c) If $\varepsilon_r = 2.5$ in the region determine the total charge induced on the shells and the capacitance of the capacitor.

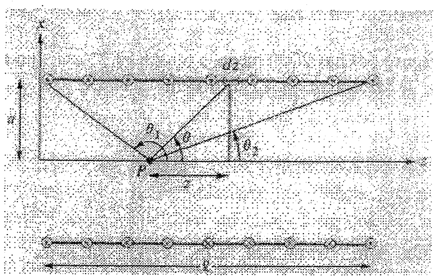
Q6.

(a) State the Biot-Savart's law.

(b) A solenoid of length l and radius a consists of N turns of wire carrying current I. Show that at point P along its axis

$$\boldsymbol{H} = \frac{nI}{2}(\cos\theta_2 - cos\theta_1)\boldsymbol{a}_z$$

were $n = {}^{N}/_{l}$, θ_1 and θ_2 are the angles subtended at P by the end turns as illustrated in the figure.



(c) Show that if $l \gg a$ at the center of the solenoid $\boldsymbol{H} = nI\ \boldsymbol{a}_z$

Section C

Select **one** question from this section. (30 Marks)

Q7.

(a) State Maxwell's equations in differential and integral froms.

(b) Derive the equation $\frac{\partial \rho}{\partial t} + \nabla . \boldsymbol{J} = 0$, using Maxwell's Equations when ρ – Charge density and $\boldsymbol{J}$ - Current density.

(c) Given the total electromagnetic energy

$$W = \frac{1}{2} \int (\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B}) dv$$

Show from Maxwell's equations that

$$\frac{\partial W}{\partial t} = -\oint_S (\boldsymbol{E} \times \boldsymbol{H}) \cdot \boldsymbol{ds} - \int_v \boldsymbol{E} \cdot \boldsymbol{J}\, dv$$

Q8.

(a) Define the Poynting vector and state the Poynting theorem.

(b) A uniform plane wave propagating in a medium has

$$\boldsymbol{E} = 2e^{-\alpha z} \sin(10^8 t - \beta z) \boldsymbol{a}_y V/m$$

If the medium is charactrized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3\ S/m$. Find α, β and $\boldsymbol{H}$.

(c) Find the Poynting vector.

Note:

1. $\nabla . (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} . \nabla \times \mathbf{A} - \mathbf{A} . \nabla \times \mathbf{B}$
2. $\nabla \times (\nabla V) = 0$
3. $\mu_0 = 4\pi \times 10^{-7} H/m$
4. $\varepsilon_0 = \frac{10^{-9}}{36\pi} F/m$

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