# THE OPEN UNIVERSITY OF SRI LANKA

## DEPARTMENT OF PHYSICS

## BACHELOR OF SCIENCE DEGREE PROGRAMME -2012/2013

## PYU2160 – MODERN PHYSICS - LEVEL 04 - FINAL EXAMINATION

**TIME: TWO HOURS (2 hrs)** **ANSWER FOUR QUESTIONS ONLY**

Date: 18.12.2013 Time: 9.30 am – 11.30 am

You may assume that, mass of electron $m_e$ = 9.1 x $10^{-31}$ kg, $h$ = 6.63 x $10^{-34}$ J s, $\pi$ = 3.14, $\hbar$ = 1.05 x $10^{-34}$ J s, $c$ = 3 x $10^8$ m $s^{-1}$, 1 eV = 1.6 x $10^{-19}$ J, $R_H$ = 1.0968 x $10^7$ $m^{-1}$.

1) The Planck law for the intensity distribution of radiation from a blackbody is written:

$$I(\lambda)=\frac{2\pi hc^2}{\lambda^5\left(e^{\left(hc/\lambda kT\right)}-1\right)}$$

where $h$ is Planck's constant, $c$ is the speed of light, $k$ is Boltzmann's constant, $T$ is the absolute temperature of the blackbody, and $\lambda$ is wavelength.

The Rayleigh radiation law is expressed as:

$$I(\lambda)=\frac{2\pi ckT}{\lambda^4}$$

(i) Sketch both laws (Planck law and Rayleigh radiation law) as a function of wavelength at a given temperature (your plot only needs to be qualitatively correct).

**(5 marks)**

(ii) Show that the Planck law reduces to the Rayleigh radiation law at very large wavelengths $\left(\lambda >> \frac{hc}{kT}\right)$.

You may need the Taylor series expansion of the exponential function which is expressed as:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots\ldots\ldots\ldots$$

**(5 marks)**

(iii) Planck's constant does not appear in the Rayleigh law, but does appear in the Planck law. Briefly explain why, in terms of the physical origin of blackbody radiation.

**(5 marks)**

(iv) On a single diagram, sketch the Planck law for the intensity distribution $I(\lambda)$ versus $\lambda$ for two temperatures $T_1$ and $T_2$ such that $T_1 < T_2$. State the Wien displacement law and with reference to your diagram explain what it means.

**(5 marks)**

(v) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C. In which region of spectrum this radiation belongs to?

**(5 marks)**

2) An 'atom laser' is made by bunching atoms together so that they have closely similar momenta and positions. This is usually done by confining the atoms laterally and slowing them down to a common velocity in the remaining dimension. One way of slowing atoms down is to arrange for them to absorb photons.

(i) Write down an expression for the momentum $p$ of a photon of wavelength $\lambda$ in terms of $\lambda$ and Planck's constant $h$.

**(4 marks)**

(ii) Explain what is meant by the Balmer series of spectral lines of hydrogen and calculate the wavelength of the longest wavelength spectral line in this series using the Rydberg formula

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_{lower}^2 - n_{upper}^2} \right)$$ **(8 marks)**

(iii) Calculate the change in velocity of a hydrogen atom moving at 2000 m/s if it absorbs a photon travelling in the opposite direction and having the same wavelength as the longest wavelength in the Balmer series.

**(8 marks)**

(iv) Explain why the photon would need to have a slightly shorter wavelength than the answer to (iii) if it is to be absorbed by the atom.

**(5 marks)**

3) A quantum particle of mass $m$ moves in a potential well of length $2L$. Its potential energy is infinite for $x < -L$ and for $x > +L$. Inside the region $-L < x < +L$, its potential energy is given by

$$U(x) = -\frac{\hbar^2 x^2}{mL^2\left(L^2 - x^2\right)}$$

In addition, the particle is in a stationary state that is described by the wave function $\psi(x) = A\left(1 - \frac{x^2}{L^2}\right)$ for $-L < x < +L$ and by $\psi(x) = 0$ elsewhere.

(i) Write down the 1-D time-independent Schrödinger equation to describe a particle with mass $m$ and energy $E$ subject to a potential $U(x)$.

**(3 marks)**

(ii) Determine the energy $E$ of the particle in terms of $\hbar$, $m$, and $L$.

**(5 marks)**

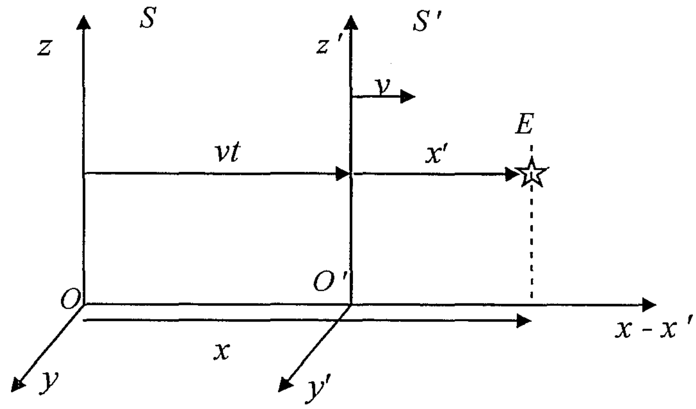
(iii) Using the normalizing condition show that

$$A = \left(\frac{15}{16L}\right)^{\frac{1}{2}}$$ **(5marks)**

(iv) Determine the expectation value of $x$. **(6 marks)**

(v) Determine the probability that the particle is located between $x < -\frac{L}{3}$ and $x < +\frac{L}{3}$. **(6 marks)**

4) Let us consider two different inertial frames of reference $S$ and $S'$ in which S ' is moving with uniform velocity $v$ relative to S along $x$ - $x'$ axes, as represented in the figure given below.




(i) In classical mechanics the coordinates $(x, y, z, t)$ and $(x', y', z', t')$ of a point $E$ with respect to two frames are related by the standard Galileo transformation $x' = x - vt$, $y' = y, z' = z, t' = t$ (in a general vectorial form they can be written as $\vec{r}' = \vec{r} - \vec{v}t$, $t' = t$). Consider a particle moving at speed $V$ along the $x$ – axis in $S$. What is its speed $V'$ in $S'$?

**(7 marks)**

(ii) In special relativity the coordinates $(x, y, z, t)$ and $(x', y', z', t')$ of a point $E$ with respect to two frames are related by the Lorentz transformations $x' = \gamma(x - vt)$, $y' = y$, $z' = z$, and $t' = \gamma\left(t - \frac{vx}{c^2}\right)$. Consider a particle moving in both reference frames S and S′. The components of the velocities of the particle with respect to two frames are given as follows:

in S, $\vec{V} \equiv (V_1, V_2, V_3) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$ and

in S′, $\vec{V}' \equiv (V_1', V_2', V_3') = \left(\frac{dx'}{dt'}, \frac{dy'}{dt'}, \frac{dz'}{dt'}\right)$.

Show that the law of the relativistic velocity transformations as

$$V_1' = \frac{V_1 - v}{1 - \frac{vV_1}{c^2}}, \qquad V_2' = \frac{V_2}{\gamma\left(1 - \frac{vV_1}{c^2}\right)}, \qquad V_3' = \frac{V_3}{\gamma\left(1 - \frac{vV_1}{c^2}\right)}.$$

**(9 marks)**

(iii) Let's assume that the velocity of the particle in the two frames is collinear, e.g. $(V, 0, 0)$ and $(V', 0, 0)$. Let $v = 0.9c$. In S' the particle moves at a speed of $V' = 0.8c$. What is the speed of the particle in the S frame, as determined according to the classical mechanics and also according to special relativity? What happens if $v = V' = c$?

**(9 marks)**

5) An important physical effect, called the Doppler effect, is experienced in everyday life on sound waves. When an ambulance passes us, the siren appears to change its note. As it approaches, the pitch is high. As it passes us, the pitch falls. The effect occurs with any type of waves, including light, and has many scientific applications.

Consider a source of radiation moving with a relativistic velocity $V$ relative to an observer. We take S as being the frame of the observer and S′ to be the frame commoving with the source. The velocity of the source can be decomposed in two components at right angles, viz. $V_r \equiv$ radial component of the velocity and $V_T \equiv$ transverse component of the velocity, where $V^2 = V_r^2 + V_T^2$. In S', the sources emits pulses with frequency $\nu_0 = \frac{1}{T_0}$, where $T_0$ is the time between pulses. In S the pulses are received with a frequency $\nu = \frac{1}{T}$, where $T$ is the time between pulses.

(i) Find the relation between the arrival of the first and of the second pulse in S. **(5 marks)**

(ii) The Lorentz transformation tells us that a moving source suffers time dilation. Find the expression of $T$ in the frame $S$. **(5 marks)**

Show the ratio of the frequencies of the radiation emitted by the source in the two frames as $\frac{\nu_0}{\nu} = \frac{1 + \frac{V_r}{c}}{\sqrt{1 - \frac{V^2}{c^2}}}$. **(5 marks)**

(iii) Show the formula for the ratio of the frequencies in the two frames (the Doppler effect formula) in the case of a source having only a radial component of the velocity as $\frac{\nu_0}{\nu} = \sqrt{\frac{1 + \frac{V_r}{c}}{1 - \frac{V_r}{c}}}$. **(5 marks)**

(iv) Find the expression for the relativistic Doppler effect in the case of a source having only a tangential component of the velocity. **(5 marks)**

6) (a) Without deriving any equation state the relativistic expression of mass – velocity relation for a particle of rest mass $m_0$. Write down the relativistic expressions for its momentum $p$ and its total energy $E$. Hence or otherwise, prove that $E^2 - p^2c^2 = m_o^2c^4$. **(8 marks)**

(b) Consider a particle of rest mass $m_0$ travelling so that its total energy is just thrice (three times) its rest mass energy. If it collides with a stationary particle of rest mass $m_o$ to form a new particle, determine the rest mass of the new particle in terms of $m_0$. **(9 marks)**

(c) A body at rest spontaneously breaks up in to two parts which move in opposite directions to each other. The parts have rest masses of 4 kg and 6 kg and have speeds of 0.8c and 0.6c respectively. Determine the rest mass of the original body. **(8 marks)**

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