The Open University of Sri Lanka
B.Sc. Degree Programme- Level 04
Final Examination 2012/2013
PHU 2142/PHE 4142- Advanced Electromagnetism

Duration: Two (2) Hours

Date: 27.11.2013 **Time: 09.30 a.m. – 11.30 a.m.**

($\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} and $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})

ANSWER FOUR QUESTIONS ONLY.

1. Let A and B be vector functions of (x, y, z) and let Φ be a scalar function of (x, y, z).

(a) How do you write the divergence of a curl of vector A and curl of gradient of scalar Φ with the ∇ operator?

(b) Prove that the divergence of a curl of a vector and the curl of a gradient of a scalar is always zero.

(c) What does the expression (**A**.∇) **B** mean in terms of the Cartesian components?

(d) Evaluate $(\bar{r}.\nabla)\bar{r}$, where $\bar{r} = \dfrac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}}$.

2. The electric field of a point charge q is given by $E = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r}$.

(a) Calculate $\square \times \boldsymbol{E}$ for a point charge.

(b) Show that the integral around any closed path is zero for a point charge.

(c) Evaluate $\square \times \boldsymbol{E}$ from (c) by applying Stokes' theorem.

(d) How do you conclude from (a) or (d) that $\square \times \boldsymbol{E} = 0$, for any static charge distribution?

3. A disk of radius r carries a uniform surface charge density σ. The total charge, Q on the disk is $Q = \pi r^2 \sigma$. The z – axis passes through the centre of the disk perpendicularly.

(a) Write down an expression for the electric field at a distance d on the axis (say z- axis) of a ring carrying a charge q.

(b) Hence find out the electric field $\boldsymbol{E}$ (magnitude and direction) at the point P, at a distance d above the centre of the disk. Express your answer $\boldsymbol{E}$ (z) in terms of Q, r, ε_o and d.

(c) Plot $\boldsymbol{E}$ (z) as a function of z for all positive z values. Use r as your unit on the abscissa and use $Q / (4\pi\varepsilon o r^2)$ as your unit for $\boldsymbol{E}$ (z).

(d) Using Gauss's law, calculate $\boldsymbol{E}$ (z) near the centre of the disk, assuming $d << r$.

4. (a) State the Biot-Savart law which gives the magnetic field produced by a current element at a distance r from the element.

(b) Using Biot-Savart law, find the magnetic field at a distance r from a long straight wire carrying a current I.

(c) Also find the magnetic field at a distance B along the axis of a circular current loop of radius a, that carries a current I.

(d) A circular coil of radius 5 cm has 10 turns and carries a current of 5 amperes. Find the magnetic field at the centre of the coil.

5. A long straight solid cylindrical conducting wire with radius R carries a steady current I.

(a) Calculate the magnetic field energy inside a length l of the wire.

(b) What is the contribution of the interior portion of the conductor to the total self-inductance?

(c) A coil with resistance of 0.05 Ω and self-inductance 0.09 H is connected across a 12 V battery of negligible internal resistance. How long after closing the switch the current in the coil reach 95 percent of its final value?

(d) At that time how much energy (in Joules) is stored in the magnetic field and how much energy has been delivered by the battery?

6. (a) Write down Maxwells equations in free space.

(b) Explain what is meant by displacement curret and identify it in the above equations.

(c) By considering Ampers law and the magnetic field in the vicinity of a charging capacitor, show that there is a need to introduce displacement current.

(d) Show that the displacement current is essential for the derivation of propagation of electromagnetic waves in free space by demonstrating E obays the wave equation $\nabla^2 E - \varepsilon_0 \mu_0 \frac{\partial E^2}{\partial t^2} = 0$.