The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination- 2013/2014
Applied Mathematics-Level 05
APU3150/AMU3181/AME5181 – Fluid Mechanics

Duration:-Two hours

Date:-17.11.2014 Time:-9:30a.m.-11:30a.m.

Answer **FOUR** questions only.

1. Obtain the **equation of continuity** in the form $div\,\underline{q} = 0$ for a perfect incompressible fluid, $\underline{q}$ being the velocity.

   Express this equation fluid in terms of **spherical polar coordinates** (r, θ, ω).

   Fluid velocity, at a point having spherical polar coordinates (r, θ, ω) has components given by $\underline{q} = \left(\frac{2k\cos\theta}{r^3}, \frac{k\sin\theta}{r^3}, 0\right)$, where k is a constant.

   (i) Show that this represents a possible motion of an incompressible fluid, and find the equations of streamlines.

   (ii) Verify that the motion is irrotational, and find the velocity potential ϕ_1.

   (iii) If another velocity potential $\phi_2 = Ur\cos\theta$ is added to ϕ_1, find the value of a, in terms of U and k so that in the combined motion there is no volume flux across the sphere $r = a$.

2. In terms of **cylindrical polar coordinates** (r, θ, z), the velocity potential Φ in an irrotational motion of an incompressible fluid is given by $\Phi = U\left(r + \frac{a^2}{r}\right)\cos\theta$, $r \geq a$, where U and a are positive constants. **Derive** the velocity at any point in the region of flow, in component form $\mathbf{q} = q_r\mathbf{e}_r + q_\theta\mathbf{e}_\theta$, and show that the equation of continuity is satisfied by this velocity.

   Obtain the equations of the streamlines in the form $\psi(r, \theta) = b$ and $z = c$, where the function $\psi(r, \theta)$ is to be determined and b, c are arbitrary constants.

   Verify further that (i) $(grad\Phi)\cdot(grad\psi) = 0$ and (ii) $\nabla^2\psi = 0$ and locate the points of **stagnation**.

3. A sphere, whose radius at time t is $R(t)$, with center O fixed, vibrates radially in an infinite incompressible liquid of constant density ρ which occupies the region $r \geq a$.

Verify that the motion of the liquid is irrotational with velocity potential $\phi(r,t) = \frac{R^2}{r}\left(\frac{dR}{dt}\right)$, where r is the distance of any point P in the liquid measured from O. The liquid is under the action of **no external body forces**. It extends to infinity, where it is at rest and the pressure there is p_∞. Assuming ***Bernoulli's equation*** show that the pressure at the surface of the sphere $(r = R)$ at time t is $p = p_\infty + \rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right)$.

If $R = a(2 + \cos\omega t)$, show that pressure on the surface $r = R$ is non-negative, if $p_\infty \geq 3\rho a^2\omega^2$.

4. A uniform solid sphere of mass M and radius a moves in a straight line with velocity V through an infinite liquid which is at rest at infinity where the pressure is p_∞.

Obtain the velocity potential in the form $\phi = \frac{Va^3}{2r^2}\cos\theta$, where (r,θ,ω) denote suitably defined **spherical polar coordinates**, and establish the following results:

(i) The liquid in contact with points on the great circle of the surface whose plane is perpendicular to the direction of motion at infinity has a velocity $\frac{1}{2}V$, relative to the sphere.

(ii) The kinetic energy of the liquid is $\frac{1}{4}M'V^2$, where M' is the mass of the liquid displaced by the sphere.

(iii) The force acting on the sphere is $F = \left(M + \frac{1}{2}M'\right)\frac{dV}{dt}$.

5. Obtain the relationship between the *velocity potential* ϕ and the *stream function* ψ representing the same two-dimensional fluid motion, stating the conditions under which they exist.

Verify that the velocity potential $\phi = -m \log\left(\frac{r_1}{r_2}\right)$, where $r_1^2 = (x-a)^2 + y^2$ and $r_2^2 = (x+a)^2 + y^2$, m and a being constants, represents a possible fluid motion.

Find the *components of velocity* and show that,

(i) the fluid speed, $q = \frac{2am}{r_1 r_2}$ and

(ii) the stream function may be expressed in the form, $\psi = -m \tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$.

Hence, show that the streamlines belong to a family of co-axial circles.

6. A two-dimensional source of strength m is placed at the point $z = c$, outside of a circular boundary $|z| = a$. Using the *circle theorem* of Milne-Thomson, write down the complex potential, and identify the image system.

By direct calculations using the image system or otherwise, show that, at any point on the boundary,

(i) the radial component of velocity is zero,

(ii) the tangential component of velocity is q, where $q = \frac{2mc\sin\theta}{a^2 + c^2 - 2ac\cos\theta}$.

Locate the points of minimum pressure on the boundary.
