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The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2013/2014
Applied Mathematics
AMU2184/ AME4184 – Newtonian Mechanics
Duration :- Two Hours

Date :-21.11.2014 **Time:-02.00 p.m.-04.00 p.m.**

Answer Four Questions Only.

1. A particle moving in a straight line, is subject to a retardation of kv^n where v is the speed at time t and n is a positive constant. Find v as a function of t.

   Show that, if $n<1$, particle will come to rest at a distance $\frac{u^{2-n}}{k(2-n)}$ from the point of projection at a time $\frac{u^{1-n}}{k(1-n)}$ where u is the initial speed.

   What happens when

   (i) $1<n<2$? (ii) $n>2$?

2. (i) In a usual notation show that the velocity and acceleration components in plane polar coordinates of a particle moving in a plane are given by $\underline{v}=\dot{r}\underline{e}_r+r\dot{\theta}\underline{e}_\theta$ and

   $$\underline{a}=(\ddot{r}-r\dot{\theta}^2)\underline{e}_r+\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta .$$

   (ii) Two particles A and B, each of mass m, are attached to the ends of a light inextensible string of length $2a$. The string passes through a smooth ring fixed at the point O on a smooth horizontal table. Initially the particles lie at rest on the table with $OA = OB = a$ and AOB a straight line l. Particle A is projected with speed u perpendicular to OA. If (r, θ) are the polar coordinates of A at time t relative to pole as the pole and the initial line l, show that:

   (a) $2\frac{d^2r}{dt^2}-\frac{a^2u^2}{r^3}=0$ (b) $2r\frac{dr}{dt}=u\sqrt{2(r^2-a^2)}$ (c) $r^2=a^2+\frac{1}{2}u^2t^2$.

3. (i) With the usual notation show that the equation of the orbit of a particle subjected to a central force is given by $\frac{d^2u}{d\theta^2}+u=\frac{F}{h^2u^2}$ with $\dot{\theta}=hu^2$.

(ii) At time t the polar coordinates of a particle of unit mass moving in a plane are (r, θ). The only force acting on the particle is $\underline{F} = \frac{\mu}{r^3}\underline{e}_r$, where μ is a constant and $\underline{e}_r$ is the unit vector along the radial direction. Obtain the equations of the possible paths.

4. (i) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A rocket-driven car moves along a straight horizontal road. The car has total initial mass M. It propels itself forwards by ejecting mass backwards at a constant rate λ per unit time at a constant speed U relative to the car. The car starts from rest at time $t = 0$. At time t the speed of the car is v. The total resistance to motion is modelled as having magnitude kv, where k is a constant.

Given that $t < \frac{M}{\lambda}$, show that

(a) $\frac{dv}{dt} = \frac{\lambda U - kv}{M - \lambda t}$.

(b) $v = \frac{\lambda U}{k}\left\{1 - \left(1 - \frac{\lambda t}{M}\right)^{\frac{k}{\lambda}}\right\}$.

5. A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the string are attached to fixed points on the same horizontal level $AB = 2l$. the mid-point of AB is C. The ball hangs in equilibrium at a distance $\frac{3l}{4}$ vertically below C.

(i) Show that $k = 10$.

(ii) The ball is now pulled vertically downwards until it is at a distance $\frac{12l}{5}$ below C and it is released from rest. Find the speed of the ball as it reaches C.

6. A particle is released from rest at a height h above the surface of the earth. Show that it will hit the earth with a speed equal to $\sqrt{\frac{2GMH}{R(R+H)}}$ where R is the radius and M the mass of the earth. Find an expression for the time of fall.