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The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2013/2014
Applied Mathematics
APU2142/ APE4142 – Newtonian Mechanics I

Duration :- Two Hours

Date :-21.11.2014 **Time:-02.00 p.m.-04.00 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation show that the velocity and acceleration components in intrinsic coordinates for a particle moving in a plane are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$ where $\underline{t}$ is the unit vector in the direction of the tangent and $\underline{n}$ is the unit vector in the direction of the inward normal.

(ii) A smooth wire in the form of an arch of the cycloid, with intrinsic equation $s = 4a\sin\psi$, is fixed in a vertical plane with its vertex downwards. The tangent at the vertex is horizontal. A small bead, of mass *m*, is threaded onto the wire and is subject to an air resistance of magnitude $\frac{mv^2}{8a}$ when its speed is *v*, this resistance being always directly opposite to the direction of motion. Given that the bead is projected from the vertex with speed $\sqrt{8ag}$, show that the bead comes to instantaneous rest at a cusp where $s = 4a$.

2. (i) With the usual notation show that the velocity and acceleration components in plane polar coordinates of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$.

(ii) Two particles *A* and *B*, each of mass *m*, are attached to the ends of a light inextensible string of length *2a*. The string passes through a smooth ring fixed at the point *O* on a smooth horizontal table. Initially the particles lie at rest on the table with $OA = OB = a$ and AOB a straight line *l*. Particle *A* is projected with speed *u* perpendicular to *OA*. If (r, θ) are the polar coordinates of *A* at time *t* relative to *O* as the pole and *l* as the initial line, show that:

(a) $2\frac{d^2r}{dt^2} - \frac{a^2u^2}{r^3} = 0$ (b) $2r\frac{dr}{dt} = u\sqrt{2(r^2 - a^2)}$ (c) $r^2 = a^2 + \frac{1}{2}u^2t^2$.

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3. With the usual notation show that the equation of the central orbit is given by

$$\frac{d^2u}{d\theta^2}+u=\frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta}=hu^2.$$

At time t the polar coordinates of a particle of unit mass moving in a plane are (r, θ). The only force acting on the particle is $\underline{F}=\frac{\mu}{r^3}\underline{e}_r$, where μ is a constant and $\underline{e}_r$ is the unit vector along the radial direction. Obtain the equations of the possible paths.

4. (i) Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}+\underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A rocket-driven car moves along a straight horizontal road. The car has total initial mass M. It propels itself forwards by ejecting mass backwards at a constant rate λ per unit time at a constant speed U relative to the car. The car starts from rest at time $t = 0$. At time t the speed of the car is v. The total resistance to motion is modelled as having magnitude kv, where k is a constant.

Given that $t<\frac{M}{\lambda}$, show that

(a) $\frac{dv}{dt}=\frac{\lambda U-kv}{M-\lambda t}$.

(b) $v=\frac{\lambda U}{k}\left\{1-\left(1-\frac{\lambda t}{M}\right)^{\frac{k}{\lambda}}\right\}$.

5. **(i)** For a system of particles subjected to impulsive forces, show that the impulsive moment of the resultant force about the axis is equal to the gain of angular momentum about the same axis.

(ii)

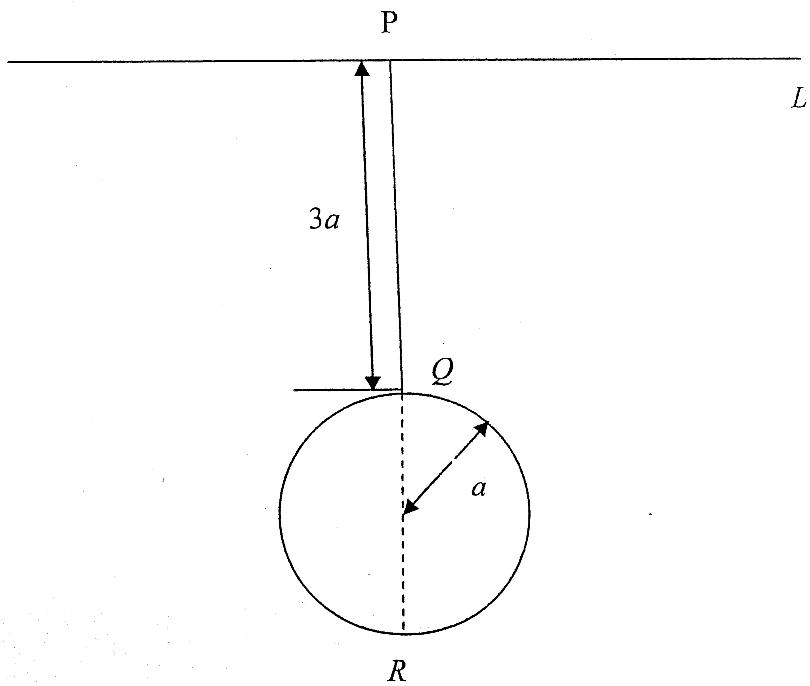

Figure 1

A thin uniform straight rod PQ has mass m and length $3a$. A thin uniform circular disc, of mass m and radius a, is attached to the rod at Q in such a way that the rod and the diameter QR of the disc are in a straight line with $PR = 5a$. The rod together with the disc form a composite body, as shown in *Figure 1*. The body is free to rotate about a fixed smooth horizontal axis L through P, which is perpendicular to PQ and in the plane of the disc.

(a) Show that the moment of inertia of the body about L is $\dfrac{77ma^2}{4}$.

(b) When PR is vertical, the body has angular speed ω and the centre of the disc strikes a stationary particle of mass $\dfrac{m}{2}$. Given that the particle adheres to the centre of the disc, find, in terms of ω, the angular speed of the body immediately after the impact.

6. (i) A uniform square lamina $ABCD$, of mass m and side $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the lamina. Show that the moment of inertia of the lamina about L is $\frac{8ma^2}{3}$.

(ii) Given that the lamina is released from rest when the line AC makes an angle $\frac{\pi}{3}$ with the downward vertical, find the vertical component of the force acting on the lamina at A when the line AC is vertical.

(iii) Given instead that the lamina now makes small oscillations about its position of stable equilibrium, find the period of those oscillations.