THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF SCIENCE DEGREE PROGRAMME -2013/2014

PYU2160 – MODERN PHYSICS

FINAL EXAMINATION- LEVEL 04

TIME: TWO HOURS (2 hrs) **ANSWER FOUR QUESTIONS ONLY**

Date: 01.12.2014 Time: 9.30 am – 11.30 am

You may assume that, mass of electron m_e = 9.1 x 10^{-31} kg, h = 6.63 x 10^{-34} J s, π = 3.14, $\hbar$ = 1.05 x 10^{-34} J s, c = 3 x 10^8 m s^{-1}, 1 eV = 1.6 x 10^{-19} J, R_H = 1.0968 x 10^7 m^{-1}.

1) (a) An X ray photon of wavelength λ recoils (Compton scatters) with increased wavelength λ' from a stationary electron of mass m and at an angle ϕ with respect to its initial direction. Show that the change in wavelength is

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi).$$

(b) Show that the maximum kinetic energy E_k, called the Compton edge, that a recoiling electron can carry away from a Compton scattering event is given by

$$E_k = \frac{hc}{\lambda\left(1+\dfrac{mc\lambda}{2h}\right)}.$$

(c) The maximum kinetic energy given to the electron in a Compton scattering event plays a role in the measurement of gamma-ray spectra using scintillation detectors. Suppose that the Compton edge in a particular experiment is found to be 520 keV. What were the wavelength and energy of the incident gamma rays?

2) (a) In the Bohr model of the hydrogen atom, an electron is orbiting around a proton such that its angular momentum $mvr = n\hbar$ (where n is an integer 1,2,3………) and its kinetic energy $KE = k_e \dfrac{e^2}{2r}$ (where $k_e = \dfrac{1}{4\pi\varepsilon_0} = 8.9\times10^9$ N m^2 C^{-2}).

(i) Show that the radius of a Bohr orbit is given as $r_n = \dfrac{n^2\hbar^2}{mk_e e^2}$

(ii) If the total energy of the atom E is given by $E = -k_e \dfrac{e^2}{2r}$, and Bohr radius $a_0 = \dfrac{\hbar^2}{mk_e e^2}$, then show that the energy of the electron in the hydrogen atom is given by

$$E_n = -\frac{1}{n^2}\frac{k_e e^2}{2a_0}$$

(iii) Calculate the ionisation energy for the hydrogen atom in eV.

(b) Hydrogen gas is put in a glass tube at low pressure, a potential difference is applied between the ends, and an electric current is passed through the gas. The radiation emitted is analysed by a diffraction grating spectrometer and the wavelengths of the emitted spectral lines are 656.3 nm (red), 486.1 nm (green), 434.1 nm (blue) and 410.2 nm (violet).

(i) Using an energy level diagram, show the origins of these spectral lines.

(ii) A further line was discovered at 122 nm. Calculate the energy of this line, and determine its origin.

(iii) The potential difference across the glass tube is gradually reduced. Although the 122 nm line remains, the red, green, blue and violet lines begin to disappear. Why?

3) A particle of mass m is in an infinite square well potential given by

$$V = \infty \qquad x < -\frac{L}{2}$$

$$V = 0 \qquad -\frac{L}{2} < x < +\frac{L}{2}$$

$$V = \infty \qquad x > +\frac{L}{2}$$

Since this potential is symmetric about the origin, the probability density $|\psi(x)|^2$ must also be symmetric. Here $\psi(x)$ is the solution of the time-independent Schrödinger equation.

(a) Show that this implies that either $\psi(-x) = \psi(x)$ or $\psi(-x) = -\psi(x)$.

(b) Show that the proper solutions of the time-independent Schrödinger equation can be written as

$$\psi(x) = \sqrt{\frac{2}{L}}\cos\frac{n\pi x}{L} \qquad n = 1,3,5,7......$$

and

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \qquad n = 2,4,6,8......$$

(c) Show that the allowed energies given by

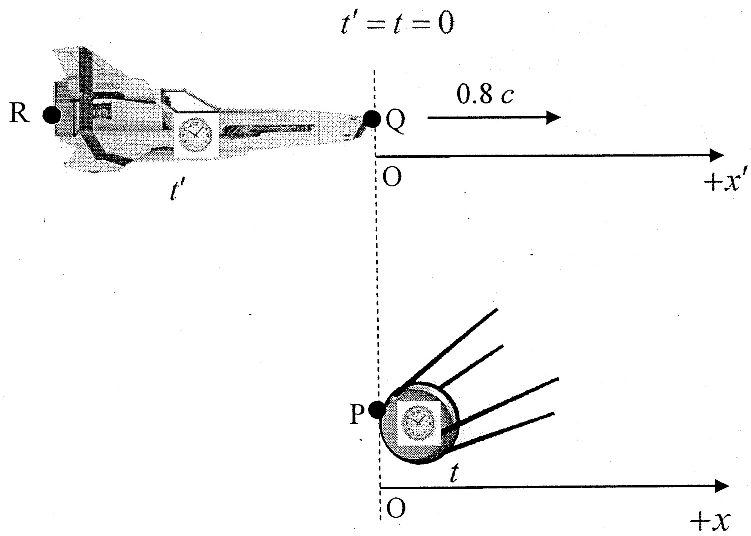
$$E_n = n^2\frac{\pi^2\hbar^2}{2mL^2} \qquad n = 1,2,3,......$$

4) (a) (i) Explain clearly the meaning of time dilation and length contraction in special relativity.

(ii) Write down the Lorentz transformations with usual notations

(iii) Starting from the Lorentz transformations between two inertial frames, derive the expression for time dilation and length contraction.

(b) Consider a spaceship (frame S') of length 50 m travels at 0.8 c passes a satellite (Frame S) at a point P shown in the figure. Clocks in the spaceship and the satellite are synchronised within their respective frames of reference and are set to zero so that $t' = t = 0$ at the instant the front of the spaceship Q passes point P on the satellite at $x' = x = 0$. At this time a light flashes at point Q.



(i) Determine the length of the ship as measured by an observer on the satellite.

(ii) Determine the time that the observer on the satellite read from the clock when the trailing edge of the spaceship R passes P.

(iii) When the light flash reaches R at the rear of the spaceship, determine the time that the observer at R reads.

(iv) Determine the time that the observer on the satellite reads when flash reaches the point R at the rear of the spaceship

5) Briefly describe the red shift related to the Doppler effects in light.

If a light source and an observer approach each other with a relative speed v, derive the relationship between the frequency f' measured by the observer and the frequency f of the source measured in its rest frame.

A light source recedes from an observer with a speed v_s that is small compared with c.

(i) Show that the fractional shift in the measured wavelength is given by the approximate expression

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v_s}{c}$$

(ii) Spectroscopic measurements of light at $\lambda = 397$ nm coming from a galaxy in Ursa Major reveal a redshift of 20 nm. Determine the recessional speed of the galaxy?

6) (a) If a particle of mass m moves with a speed v, write down the relativistic expression for the mass m in terms of its rest mass m_0, its speed v and the velocity of light c.

(b) Using the relativistic expression for mass show that the kinetic energy of a particle, E_k is given by ,

$$E_k = mc^2(\gamma - 1).$$

Here the symbols have their usual meaning.

(c) A cosmic-ray proton is moving at such a speed that it can travel from the Moon to Earth in 1.5 s.

(i) At what fraction of the speed of light is the proton moving?

(ii) What is its kinetic energy?

(iii) What value would be measured for its mass by an observer in Earth's reference frame?

(iv) What percent error is made in the kinetic energy by using the classical relation?

Take the Earth-Moon distance as 3.8 x 10^5 km. Ignore Earth's rotation.

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