The Open University of Sri Lanka

B.Sc. Degree Programme-Level -04

Final Examination-2013/ 2014

AMU 2183/ AME4183-Mathematical Modelling III

Applied Mathematics Level-04

Duration: Two Hours

Date: 02-07-2014.	Time: 09.30 a.m. – 11.30 a.m.

Answer Four Questions only

1. (a) A private phone vendor plans to purchase a total of 29 Nokia and Ericsson phones. The cost of a Nokia phone is Rs.800 more than a Sony Ericsson phone. The total cost of phones is Rs.166000. If the cost of each phone is a multiple of 100, find the number of Nokia and Sony Ericsson phones the vendor can purchase.

   (b) Suppose the price of Nokia phone has gone up by Rs.100, what will be the extra cost the vendor has to bear if the vendor decides to purchase the same number of Nokia and Sony Ericsson phones as before?

2. Let $Z(t)$ be the number of infected persons in a hospital at time t. Suppose the rate at which the number of infected persons increase is proportional to the product of the number of infected persons and those not infected in the hospital. Assume that the total number of persons in the hospital is fixed and is equal to N.

   (a) Write an expression for the rate at which the number of infected persons in the hospital increase.

   (b) If Z_0 is the initial number of infected persons in the hospital, show that $Z(t)$ is given by

   $$Z(t) = \frac{N}{1 + \left(\dfrac{N}{Z_0} - 1\right)e^{-Nkt}} \text{ , where } k \text{ is a constant.}$$

   (c) Let Z_1 and Z_2 be the values of $Z(t)$ when $t = T$ and $t = 2T$ respectively. Show that

   $$N = \left(\frac{\dfrac{1}{Z_0} - \dfrac{2}{Z_1} + \dfrac{1}{Z_2}}{\dfrac{1}{Z_0 Z_2} - \dfrac{1}{Z_1^2}}\right).$$

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3. A uniform beam of length l and weight w per unit length is placed horizontally with two smooth supports at its ends. If the flexural rigidity of the beam is k, show that the beam will rest along a curve whose equation with respect to a suitable coordinate system is

$$y = \frac{wx}{24k}(l-x)\left[l^2 + x(l-x)\right].$$

Hence, find the maximum deflection of the beam.

4. Suppose you have Rs.1000 to deposit in one of the five banks, say, A, B, C, D and E. Suppose the money is to be left in the bank for five years and you want to maximize your ending balance. Bank A pays 15% simple interest, bank B pays 14% interest compounded monthly, bank C pays 13.3% interest compounded quarterly, bank D pays 13% interest compounded continuously; and bank E pays 13.5% interest compounded weekly.

Which bank pays the most and which bank pays the least?

What is the effective interest rate of each of the banks?

5. (a) A rectangular tank is to be made of galvanized iron so as to have a capacity of V liters, where V is a constant. It will be a closed tank with a lid of negligible

   thickness. The cost of the construction depends primarily on the surface area. If the cost of construction is to be minimized, what should the dimensions of the tank be in meters? You may take 1 milliliter to be 1 cm^3.

   (b) If the cost of construction per 1 cm^2 is 30 cents, what will be the

   cost of construction for a tank that holds 850 liters?

6. Assume that the rate at which radioactive nuclei decay occurs is proportional to the number of such nuclei that are present in a given sample. Suppose half of the original number of radioactive nuclei in a sample have undergone disintegration in a period of 1500 years.

   (a) What percentage of the original radioactive nuclei will remain after 4500 years?

   (b) In how many years will only one-tenth of the original number remain?

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