The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2014/2015
Applied Mathematics – Level 04

APU2142/ APE4142 – Newtonian Mechanics I

Duration :- Two Hours

Date :-24.10.2015 **Time:- 01.30p.m.-3.30 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation show that the velocity and acceleration components in intrinsic coordinates for a particle moving in a plane curve are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$ respectively, where $\underline{t}$ is the unit vector in the direction of the tangent and $\underline{n}$ is the unit vector in the direction of the inward normal.

(ii) A bead of mass m can side along a smooth wire bent into form of part of a cycloid which is held with its vertex O downwards and its axis vertical. The intrinsic equation of the part of the cycloid is $s = 4a\sin\psi,\ \ 0 \le \psi < \frac{\pi}{2}$, where a is a positive constant. The bead is projected from the point O with speed $\sqrt{3ag}$ along the wire.

(a) Find the speed of the bead at time t.

(b) Find the magnitude of the normal contact force when $\psi = \frac{\pi}{3}$.

2. (i) With the usual notation, show that the velocity and acceleration components in plane polar coordinates of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$ respectively.

(ii) The only force acting on a body which is of mass m and is at a distance r from the centre of the Earth, is directed towards the centre of the Earth and is of magnitude $\frac{\mu m}{r^2}$, where μ is a constant.

Show that the speed of a satellite of mass m moving in a circular orbit of radius a about the centre of the Earth is $\sqrt{\frac{\mu}{a}}$

A second satellite of mass $3m$ is moving in the same circular orbit as the first but in the opposite direction and the two satellites collide and coalesce to form a single composite body. Show that the subsequent motion of the composite body is governed by the two equations: $r^2\dot{\theta} = \sqrt{\frac{a\mu}{4}}$, $\ddot{r} = \frac{\mu(a-4r)}{4r^3}$ where (r, θ) are the polar coordinates of the body with the centre of the Earth as pole. Find the values of r when $\dot{r}^2 = 0$.

3. (i) (a) With the usual notation show that the equation of a central orbit is given by

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta} = hu^2.$$

(b) A particle P moves in a path with polar equation $r = \frac{2a}{2+\cos\theta}$ with respect to a pole O and initial line OA. At any time t during the motion $r^2\dot{\theta} = h$ (constant). Determine the central force.

(ii) A particle is released from rest from a point at a height H above the surface of the earth. Show that it will hit the earth with a speed equal to $\sqrt{\frac{2GMH}{R(R+H)}}$ where R is the radius, M the mass of the earth and G is the universal constant of gravitation.

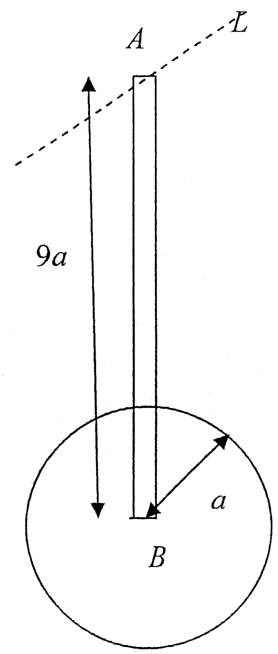
4. (i) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A body consists of equal masses M of inflammable and non-inflammable material. The body descends freely under gravity from rest. The combustible parts burns at a constant rate of kM per second, where k is a constant. The burning material is ejected vertically upwards with constant speed u relative to the body, and air resistance may be neglected. Show, that the equation of motion is given by

$$\frac{d}{dt}\left[(2-kt)v\right] = k(u-v) + g(2-kt) \text{ where } v \text{ is the speed of the body at time } t.$$

Hence, show that the body descends a distance $\frac{g}{2k^2}+\frac{u}{k}(1-\ln 2)$ before all the inflammable material is burnt.

5. A pendulum P is modelled as a uniform rod AB of length $9a$ and mass m, rigidly fixed to a uniform circular disc of mass $2m$ and radius a. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in in the following figure. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L, which passes through the end A and is perpendicular to the plane of the disc.



(a)Show that the moment of inertia of the body about L is $190ma^2$.

(b) The pendulum makes small oscillations about L. Find the period of these small oscillations.

6. A uniform square lamina $ABCD$ of mass $2m$ and side $3\sqrt{2}a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the lamina.

(i) Show that the moment of inertia of the lamina about L is $24ma^2$.

(ii) The lamina is at rest with C vertically above A. At time $t = 0$ the lamina is slightly displaced. At time t the lamina has rotated through an angle θ.

Show that $2a\dot{\theta}^2 = g(1 - \cos\theta)$.

(iii) Show that, at time t, the magnitude of the component of the force acting on the lamina at A, in a direction perpendicular to AC, is $\frac{1}{2}mg\sin\theta$.

(iv) When the lamina reaches the position with C vertically below A, it receives an impulse which acts at C, in the plane of the lamina and in a direction which is perpendicular to the line AC. As a result of this impulse the lamina is brought immediately to rest. Find the magnitude of the impulse.